MODEL OF COLLABORATION BETWEEN SUPPLIER AND MANUFACTURER FOR THE PROFIT OF SUPPLY CHAIN

by

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Dissertation submitted in partial fulfilment of

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CERTIFICATION OF APPROVAL

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Approved by,

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CERTIFICATION OF ORIGINALITY

This is to verify that I am responsible for the work submitted in this project, that the original work is own my own except as specified in the references and acknowledgement, that the original work contained herein have not been undertaken or done by unspecified sources of persons.

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(MUHAMMAD RAFAEE BIN ROMENOR)

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With deepest gratitude and appreciation,

…………………………………………

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ABSTRACT

A world class supply chain management involves value-adding activities in the supplying process which to deliver the supplying service at its' optimal total cost, the least lead time and at the highest quality. At the highest level, supply chain system can be divided into two: (1) Production planning and inventory control and (2) Logistics process. The development of this model is based on the reviews on journals and compilation of surveys such as Pahl et al. (2007) and Li et al. (2010). Both surveys were discussing about recent interests of area of research about deterioration constraints in inventory. For deterioration constraint, Ghare and Schrader (1963) had introduced a common function to describe deteriorating inventory. Chakrabarty et al. (1998) extend Philips model using three parameters Weibull distribution whereas Wee (1999) using two parameters Weibull in addition by incorporating quantity discount and partial backordering. Wee et al. (2005) has developed a joint strategy for supplier and buyer under deteriorating constraint. Hou et al. (2009) also presented deterioration model using continuous discount cash flow to simulate the effect of inflation and permissible delay. Tripathi et al. (2010) extends Hou's (2009) model by developing a mathematical model for non-deteriorating model. In this project, the mathematical model is derived from inventory function of Ghare (1963) and incorporates economic constraints such inflation and permissible delay from Tripathi et al. (2010), partial derivation of quantity discount from Benton (2007), order processing lead time reduction as per Huang et al. (2010) and effect of deterioration in supplier's inventory from Wee (1999). The new model presented in this project shall determine the optimal solutions in term of minimum present value of joint total cost can be achieved under constraints as mentioned above.

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1 CHAPTER 1: INTRODUCTION

1.1 Background of study

The recent interests behind the proposed joint action of supply chain model are to investigate all reasonable factors that would contribute to increasing supply chain cost between the buyer and the supplier. Many models presented in the journals are based on single stage duplex supply chain communication to simulate the probability of a controlled scenario in real world issue.

In reality, there are a lot of input and output going on in every direction towards the buyer and supplier. To simulate the real business condition of supply chain activities with multiple buyers and suppliers would be difficult as each of strategic partnership established between them are unique and not the same. To simplify the case studies, a single vendor and a single buyer supply chain policy representing what if conditions are a lot discussed in the operational management conferences and technical journals.

There are a lot of factors that contribute to the value of supply chain policy such as inventory management and lifespan, market price, product reliability, trade currency and inflation, scrap and salvage value from manufacturing or storage process. The main interest for this project is to develop a joint supply chain model based on the factors identified in the literature review to establish a common mutual business agreement in a controlled situation where the supplier and buyer present value of total annual costs shall be modelled to justify the motivation and the future of partnership of both side heading to.

In this project, a combined approach of stochastic and deterministic mathematical analysis shall be presented in the model of collaboration between a supplier and a buyer for the profit of supply chain.

1.2 Problem statement

1.2.1 **Problem identification**

A recent study on the literature and surveys from Li (2010) and Pahl (2007) regarding operation management models indicates a lot of researchers established their area of interests in various inventory constraints to improve the supply chain network. Those analyses cover more on economic factors, deterioration, policy incentives, inventory salvation, logistics and warehouse management model.

The optimal inventory policy presented by Hoque et al. (2000) develops an inventory model with warehouse capacity and transportation constraint with unequal lot sizing problem. As in Roger's (1997) model, he developed a general policy for single supplier and single buyer production-inventory model under fixed factor to minimize the average total cost per unit time. However, Goyal (2000) has improving the solution procedure presented by Roger (1997) and achieved more feasible optimal total cost solution. Akbari Jokar et al. (2009) interested in inventory optimization for joint economic order quantity under price sensitive demand constraint. Barron (2008) developed an inventory model under production rework problem in single cycle period. Janggi et al (2007) has looked in retailer's and supplier's point of view to provide an optimal joint cost solution with permissible delay constraint. Benton et al. (2007) improved on quantity discount model with considering the overstocking risk for buyer and supplier. Tripathi (2010), Hou (2009) and Chakrabarty (1998) discussed about effect of inflation over the planning horizon. Huang (2009) developed a stepwise inventory model for supplier and buyer under lead time problem in processing order. Some of the authors mentioned above also considering deterioration effect in their inventory model. Wee (1999) also presented an optimization for inventory under deterioration, quantity discount and partial backordering constraints.

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1.3 Objectives and scope of study

1.3.1 **Objectives**

The project shall be focusing into two dependent objectives:

- To develop a deterministic mathematical model of strategic inventory planning for joint collaboration of single supplier-buyer of optimal lot sizing under asset deterioration in which under two parameter Weibull distribution with permissible delay in settling account under factor of inflation and lead time reduction on order processing decision inclusive with quantity discount policy for both parties total annual cost reduction.
- To compare the constraint sensitivity impact on cash flow of present value total annual costs and other decision factor involved.

1.3.2 **Scope of study**

The scope of study of joint supply chain model begins from going through of some reviews on production planning and deteriorating inventory study and surveys on recent trend of constraints which have been the interest to researchers to perform analysis on more detailed and controlled case.

Development of mathematical model shall be supported with some work from the journal. The assumptions and constraints used are defined in the methodology.

An example of numerical computation from the model shall be presented in the cost analysis table together with impact of constraint on present value of total cost with individual sensitivity analysis. The optimal number of lot size and number of replenishment shall be determined from the tabulation. The most significant constraint which yields the largest impact on cost reduction shall be identified.

1.4 Relevancy of project

This project shall propose an optimized and stabilized inventory model for procurement managers as a decision tool before making any joint purchasing decision and forecasts the probability of effect from the destabilization factors for both sides; supplier and buyer under the determined circumstances in which will result in equalisation of profit distribution without sacrificing the effort of costs reduction.

1.5 Feasibility of the project

The Gantt chart in 5.1.2section can be viewed. From the Gantt chart, it can be seen that this project need extension of research to cover most of important factors to be included in the model of further improvement.

The project's scope is also covering the mechanical engineering aspect – supply chain operating management has very big influence towards company's management decision, production planning and inventory control, logistics and distribution process.

2 CHAPTER 2: LITERATURE REVIEW

2.1 Economic factor: Inventory deterioration

Wee HM (1993) defined deteriorating items as items that become decayed, damaged, evaporative, expired, invalid, devaluation and anything that reduce its utilization and reliability value throughout the time. Ruxian et al. (2010) divided deteriorating item into two categories: (1) devalued because of decay, damage or expired in natural ways such as blood, medicine, vegetable, meat and fruit. (2) Devalued because of introduction to new technology or new alternatives which yields more effective towards a solutions and possess a better efficiency, practicality and cost-effective. Ghare and Schrader (1963) studies has led into a general conclusion that depletion rate of the deteriorated inventory follows a negative exponential function of time which stated as below:

$$\frac{dI(t)}{dt} + \theta I(t) = -f(t)$$

Ghare and Schrader (1963) inventory depletion and deterioration model

where Ɵ is the rate of deterioration, I(t) is inventory size and f(t) is rate of demand).

In this project, the deterioration rate is defined as constant throughout the planning horizon. The inverse of deterioration rate is proposed as planning factor as an economic strategy for procurement manager to decide the order quantity (buyer annual demand) before making any purchase policy with the supplier. Further parameter assumption for the model is defined in the methodology.

2.2 Economic factor: Trade credit

In common situation, suppliers are generally often selling goods on credit. Fluctuations in goods or material market value always lead in force to increase the production and logistics costs. When the suppliers seek repayment and find out the money receives are less than they expected, they will become reluctant to trade. Inflation can lead to loss of supplier competitiveness by imposing stricter business policy in making joint decision. If not being careful in consideration while tailoring the business partnership between the supplier and buyer, it may jeopardize the business deal and leading to further increases in costs and major loss in revenue. Either supplier or buyer may be forced to cut back supplying the material or production due to increasing price. Therefore, in designing a joint inventory policy, it is essential to include factor of inflation in the costing analysis to find out which feasible solutions will lead to mutual benefits and ultimately yield a better profit margin for both parties. Tripathi et al (2010). in concluded that as increasing the value of factor of net discount rate of inflation, the total present value of costs will decrease. This is for a fixed number of replenishment over the planning horizon.

2.3 Economic factor: Order processing cost with lead time reduction

Order processing cost or order replenishment cost has known as one of the factor that contributes to the expenditure to the company. The company needs to determine the required demand quantity in each order cycle. In ideal situation, where JIT is possibly implemented in the inventory cycle, the order processing cost becomes zero. However in real situation it does not happen in that manner. Many models discussed the order processing cost in brief by incorporating the order processing cost as a sub cost function over the total cost function. Tripathi (2010) and Wee (1997) are also considered the replenishment cost as fixed whereas Huang (2010) implemented order processing lead time reduction function in his model.

2.4 Economic incentives: Permissible delay in settling trade credit account and quantity discount for economic order quantity

Cash discount and quantity discount incentives are commonly used by supplier to encourage their client for purchasing more lot size over the purchasing strategy. It provides the buyer to leverage an economic strategy at the fewer lot price than purchase at regular cost. Huang (2010) and Tripathi (2010) has included the trade credit policy in their inventory model.

3 CHAPTER 3: METHODOLOGY

3.1 Assumptions

In derivation of mathematical formula, some assumptions are made to simplify the model:

1. Lead time between successive consignment deliveries is zero.
2. No shortages allowed.
3. No backlogging allowed.
4. The buyer does not consider reorder point as each consignment is shipped as per JIT.
5. Safety stock planning is considered only with regular purchase policy.
6. Annual demand, D and supplier production rate, R is constant. R > D
7. Relaxation on assumption 6 to simulate a case where quantity discount policy is implemented.
8. All parameters including the fixed costs and variable costs are defined.
9. The mathematical model is designed to follow the supplier and buyer inventory planning cycle as per Figure 3-1.

3.2 Notations

D	Annual demand (unit/year)	R	Production rate (unit/year)
k	Taylor series expansion parameter, k = 3	α	Deterioration scale factor
r	Net discount of inflation rate ($ fixed portion of money ratio over $/$1/year)	β	Deterioration shape factor
P	Purchase price, $/unit	I_c	Interest charged ($ fixed portion of money ratio over $/$1/year)
I_d	($ fixed portion of money ratio over $/$1/year)	h_v	Supplier holding cost, ($/unit/year)
s_v	Supplier setup cost, ($/production planning)	h_b	Buyer holding cost, ($/unit/year)
F	Fixed transportation cost, ($/shipment)	L	Time for order processing cycle,

			(order time, year)
U	Buyer order processing cost, ($/order time)	t	Trade credit period (years)
ss	Safety stock (unit)	d_k	Quantity discount schedule, $/unit
G	Actual demand (unit/year)	Z	Acceptable portion for overstock quantity, (%)
E	Normal distribution function with overstock probability	σ	Standard deviation of demand per expected order cycle, (unit)
K	Decision factor of capital investment in employing the order processing strategy, ($/planning horizon) where K > 0; K = 1, 2, 3, …		
m	Exponential smoothing constant for order processing LTR strategy. Treat the time projection for term L is stable/near constant condition.		

NOTE:

- To maintain its' consistency, the currency used as per definition is in dollar, $.
- Trade credit period, t is assumed short than inventory cycle period, T. Thus, supplier is assumed to offer 80% of permissible delay period from the cycle time.
- LTR = Lead Time Reduction, JIT = Just in Time

3.3 Formula development

3.3.1 **Depleted inventory function with deterioration formula derivation**

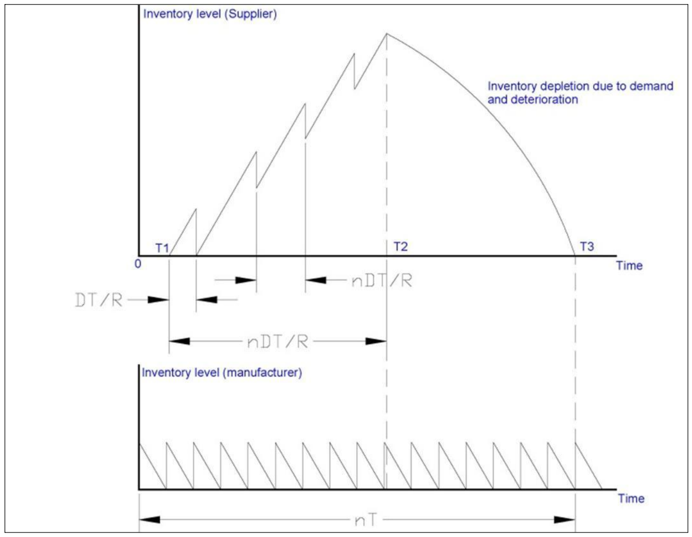


Figure 3-1Supplier and buyer inventory level with two parameter Weibull distribution

In Figure 3-1, the inventory deterioration function is expected to decrease with time. For this inventory model as described in Figure 3-1 will not consider quantity backorder and shortage as presented by Wee (1999). The item shall start deterioration from supplier's maximum inventory or economic order quantity (EOQ).

In development of inventory formula, the deterioration rate is derived from Philip (1974) generalized EOQ model with a three-parameter Weibull distribution. The probability density function *f(t)* is defined as:

$$f(t) = \propto \beta(t-\gamma)^{\beta-1} e^{-\alpha(t-\gamma)^{\beta}}$$

Chakrabarty et al. (1998) Three parameter of Weibull distribution function

where α is the scale parameter, α > 0; β is the shape parameter, β > 0; γ is the location parameter, t > γ; and t is time of deterioration, t > 0. In this model, the density function *f(t)* is reduced into two-parameter Weibull distribution because the time shifting of deterioration rate is not considered, where γ = 0; and gives:

$$f(t) = \alpha\beta t^{\beta-1}e^{-\alpha(t)^{\beta}} \text{(equation1)}$$

Weibull cumulative distribution function is defined as:

$$F(t) = 1 - e^{-\alpha(t)^{\beta}} \text{(equation2)}$$

and the rate of deterioration at time (t), Z(t) is

$$Z(t) = \frac{f(t)}{1-F(t)} \text{(equation3)}$$

Substituting (1) and (2) into (3), we get:

$$Z(t) = \alpha\beta t^{\beta-1} \text{ (equation4)}$$

The relationship between deterioration rate and time is similar to failure rate behaviour is shown in Figure 3-2below.

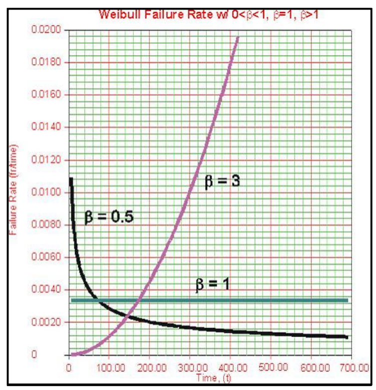


Figure 3-2Deterioration rate vs Time plot with various shape parameter.

Chakrabarty (1998) stated when $\beta < 1$, it has decreasing rate of deterioration; when $\beta > 1$, it has increasing rate of deterioration. If let $\beta = 1$, the density function yield a constant rate of deterioration.

A deteriorating inventory model developed by Ghare and Schrader (1963) follows a negative exponential function of time. The change of inventory item with deterioration rate is portrayed in the differential equation:

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t) \text{ (equation5)}$$

where Θ is the deterioration rate, *I(t)* is the inventory level at time t, and *–D(t)* is the demand rate at time t. For next derivation onwards, T shall be the domain of function. Arranging the term (4) and (5) will give:

$$\frac{dI(T)}{dt} = -\mathrm{D(T)} - \mathrm{Z(T)I(T)} \text{ (equation 6)}$$

Demand rate, *D*(t) is linearly decreasing function over time. Inventory depletion due to demand itself is presented in linear as:

$$\mathrm{D(T) = D + b(T)} \text{ (equation 7)}$$

where D is annual demand and b is demand depletion rate in unit/time. To compute cumulative inventory depletion, the succeeded negative sign in (6) is voided at first. As per assumption, the inventory depletion is directed by successful consignment delivery rather than quantity at instantaneous point. So, term t is changed to T for to maintain derivation consistency.

Integrate the term *D*(T) with respect to T at boundary condition I_{max} = I(T)gives the first term of inventory depletion without deteriorating function:

$$I_1(T) = nDT + \frac{b}{2}T^2 \text{ (equation 8)}$$

Supplier's inventory is always subjected to a step size n. So, DT in (8) is known as Q = nDT. The inventory depletion gradient, b can be illustrated as perFigure 3-3 below:

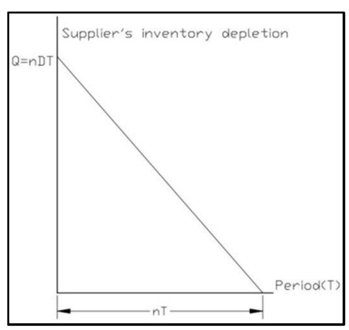


Figure 3-3 Inventory depletion due to demand rate only

Equation (8) tells the depletion due to demand rate is in form of polynomial convex function. In simplification to determine demand rate, the gradient as per Figure 3-3 can be determined as follow:

$b = \frac{-nDT}{nT}$;cancel out the common term nT and b is equal to D. In cumulative function, the succeeded negative sign is suppressed first. Equation (8) is now known as $I_1(T) = nDT + \frac{D}{2}T^2$

The inventory is not bounded by time shifting term γ. As per assumption, the deterioration rate is always follow continuous function of T. So, as per next consignment delivery, the item is expected in on-going deterioration since the supplier starts production for stocking program. To simplify the derivation work, assume the continuous time factor at instantaneous deterioration, t is treated as mean time deterioration for each consignment. Thus, *t* is always equal to T to satisfy the model.

To derive second term, further reduction two parameters of Weibull distribution at β = 1 is transformed into deterioration function of negative exponential distribution [put no of reference] , Z(t):

$$Z(T) = e^{\alpha T^{\beta}}$$

To simplify the deterioration model, the deterioration shall start from maximum cumulative inventory or supplier's economic order quantity denotes as I_{max}. I_{max} is also equals to the cumulative supplier inventory, $I_{max} = Q = nDT$.

$$Z(T)\frac{dI(T)}{dT} = Q(e^{\alpha T^{\beta}})$$

$$Z(T)\frac{dI(T)}{dT} = \int nDT(e^{\alpha T^{\beta}})dT$$

By expressing the exponential term in Taylor expansions:

$Z(T)I_2(T)$: $\int nDT \sum_{k=0}^{\infty} \frac{(\alpha T^{\beta})^k}{k!}\, dT$; $nD \sum_{k=0}^{\infty} \frac{\alpha^k}{k!} \int T.T^{\beta k}\, dT$; $nD \sum_{k=0}^{\infty} \frac{\alpha^k}{k!} \int T^{\beta k+1}\, dT$; $nD \sum_{k=0}^{\infty} \frac{\alpha^k}{k!} . \frac{T^{\beta k+2}}{\beta k+2}$; $nD \sum_{k=0}^{\infty} \frac{\alpha^k T^{\beta k+2}}{k!(\beta k+2)}$

Thus:

$$Z(T)I_2(T) = nD \sum_{k=0}^{\infty} \frac{\alpha^k T^{\beta k+2}}{k!(\beta k+2)}$$

Dividing the both side of equation with deteriorating exponential function $Z(T) = (e^{\alpha T^{\beta}})$:

$$\text{I}_2(\text{T}) = \left(nD\sum_{k=0}^{\infty}\frac{a^k T^{\beta k+2}}{k!(\beta k+2)}\right) \times e^{-\alpha T^{\beta}} \text{(equation 9)}$$

Combining (8) and (9) will give the cumulative deteriorating inventory model, I(T),

$$I(T) = nDT + \frac{D}{2}T^2 + \left(nD\sum_{k=0}^{\infty}\frac{a^k T^{\beta k+2}}{k!(\beta k+2)}\right) \times e^{-\alpha T^{\beta}} \text{ (equation 10)}$$

3.4 Supplier depleting inventory level illustration in general

From the derivation of supplier depletion inventory level formula (10), we found that it is difficult to illustrate the equation (10) in a simple plot over time, T. For simplification, assume D = 10unit/year, b = 0.1unit/time and Q = 100unit. Disregard the factors attached to the equation, the depletion inventory level can be illustrated as per equation below:

$$D(T) = 10 + 0.1T;\ I1(T) = \int D(T)dT = \int(10 + 0.1T)\ dT; I1(T) = 10T + 0.05T^2$$

(equation 11)

whereas, I2(T) = $\int I1(T) * Z(T)$, for simplification, we assume Z(T) = e^T

deriving I2(T) as $\int(100T)(e^T) = 100\int Te^T$ using integral by parts and get

$$I2(T) = 100e^T(T-1); and\ I2(T) = 100e^T(T-1) \times e^{-T} = 100(T-1)$$

(equation 12)

The solution for cumulative supplier's depleting inventory level I(T) is:

$$I(T) = I1(T) + I2(T) = (10T + 0.05T^2) + (100(T-1)) = 0.05T^2 + 110T - 100 \text{ (equation 13)}$$

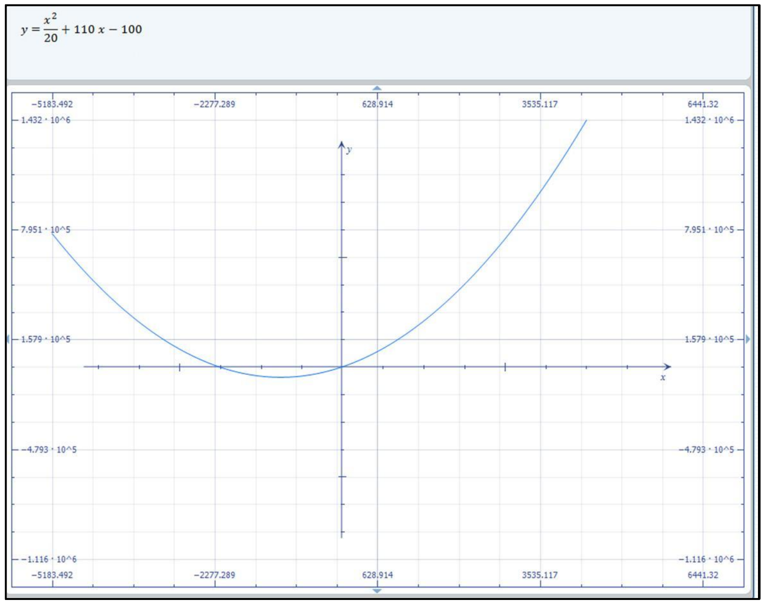


Figure 3-4 Projection of equation y(x) = (1/20)x^2 +100x – 100

The Figure 3-4shows equation (13) projection is a convex function. The projection of equation (10) should be similar like Figure 3-4. Applying the constraint where Y and X should be positive, the cumulative inventory shall start from 0 to the time of planning horizon. Carry on with the assumption in this model, the inventory depletion shall follow the proposed inventory cycle which shall be discussed in the next section.

3.4.1 **Supplier annual total cost function**

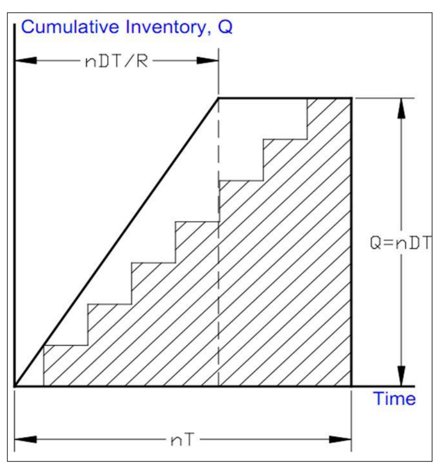


Figure 3-5 (BOLD) region is the total accumulated inventory; (HATCHED) region is the accumulated depleted inventory.

For simplification, the cumulative buyer depletion inventory level will not follow shape in Figure 3-4but in stepwise diagram Figure 3-5but still incorporate the depleting inventory function as per equation (10). Although the cumulative inventory deterioration curve is not in linear regression, the average inventory holding cost is assumed the same as deterministic EOQ model with a constant demand and without deterioration. Without this assumption, the model becomes too complex to solve.

In Figure 3-1also suggests that after some period of T, supplier will halt its production at supplier's economic lot size to allow profit return and capital recovery from sales and consignment shipping to the buyer and to allow supplier's warehouse moderation for their next production and capacity planning for the future project. The supplier net total holding cost based on inventoryvariance is given by:

Net annual holding cost = $\frac{[BOLD-HATCHED]region \times Supplier's\ holding\ cost}{nT}$

Huang (2010) Net annual holding cost function

where the total accumulated production inventory, Q(T) is equal to total accumulated depleted inventory, *I*(T) as given by:

$Q(T) = I(T)$ (equation 14)

Using the relationship above gives the net total annual cost of supplier

$TC_v(n,T)$ =[Q(T) – I(T)] x hv

= [(cumulative production inventory – Average inventory) – (cumulative inventory depletion)] x holding cost

$TC_v(n,T)$ =

$$\left\{\left[nDT\left(\frac{DT}{R}+(n-1)T\right)-\frac{nDT\left(\frac{nDT}{R}\right)}{2}\right]-\left[nDT+\frac{D}{2}T^2+\left(nD\sum_{k=0}^{\infty}\frac{a^kT^{\beta k+2}}{k!(\beta k+2)}\right)\times e^{-\alpha T^{\beta}}\right]\times\frac{hv}{nT}\right\}$$

By adding the average production setup cost, Sv; the total annual cost for supplier is:

$TC_v(n,T)$ =

$$\left\{\left[nDT\left(\frac{DT}{R}+(n-1)T\right)-\frac{nDT\left(\frac{nDT}{R}\right)}{2}\right]-\left[nDT+\frac{D}{2}T^{2}+\left(nD\sum_{k=0}^{\infty}\frac{a^{k}T^{\beta k+2}}{k!(\beta k+2)}\right)\times e^{-\alpha T^{\beta}}\right]\times\frac{hv}{nT}\right\}+\frac{Sv}{nT} \text{ (equation 15)}$$

3.4.2 **Relevant costs for buyer**

The buyer’s inventory cycle is set as per Figure 3-6is to investigate a condition where the buyer is subjected to the permissible delay cash discount incentive, t if given by the supplier is less than buyer’s settlement time, T. Assuming the supplier is playing at high risk by leveraging buyer’s solid financial capability by employing very enticing cash discount at short time before take advantage of interest earned after trade credit period which was agreed in the supply order policy. The supplier may set a single Net (t) as the trade credit period. The reason behind this is because the supplier may have extensive studies of this buyer’s previous demand and long business history on its payback competency. The strategy is that the supplier may experience profit loss in the trade credit period and for the long run, the supplier is actually multiply their profit significantly.

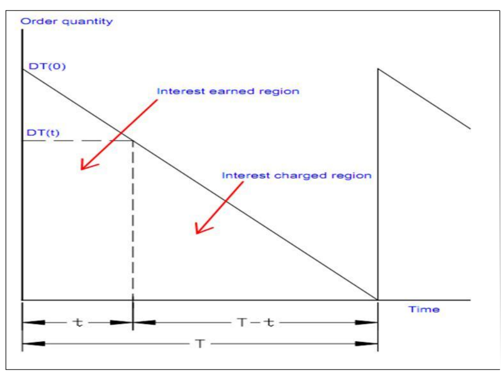


Figure 3-6 Buyer’s time dependent inventory when T > t.

The annual total costs component for buyer inventory consists of total order replenishment cost, total purchasing cost, total holding cost, total interest charges over the time horizon and total interest earned during the permissible delay settlement period. Additional cost as presented by Huang (2010) like fixed transportation cost, order processing cost and carrying cost are also considered. Arrangement of annual total costs is in similar manner as Goyal (1985).

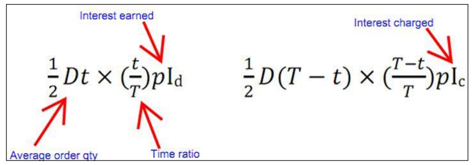


Figure 3-7 (LEFT) Value for interest earned per cycle (RIGHT) Value of interest charged per cycle

Stock level at the time of account settlement is: D(T-t). Maximum achievable interest earned during permissible delay is Dtp; if T ≥ t.

Thus, arranging the equation for total annual cost of buyer from left to right in general form is given by:

TC_b(n,T,t,K) = C1 + C2 + C3 + C4 + C5 – C6 + K; where C1 = fixed transportation cost, C2 = Order processing cost function, C3 = Buyer holding cost, C4 =Purchasing cost, C5 = Interest charge, C6 = Interest earned during permissible delay and K = Capital invested to employ the purchase order strategy.

TC_b(n,T,t,K) = $\frac{nF}{nT} + \frac{nULe^{-mK}}{nT} + \frac{D \times hb}{2} + Dp + \frac{D(T-t)\times p \times Ic}{2T} - \frac{D(t) \times p \times Id}{2T} + K$; if T ≥ t

Equation 16: Buyer annual total cost similar to Huang (2010) with integrated purchase cost

3.4.3 **Joint supplier-buyer annual total cost**

The total joint annual cost of supplier and buyer is denoted as JTC(n,T,t,K):

where JTC(n,T,t,K) = TC_v(n,T) + TC_b(n,T,t,K); if T > t(equation 17)

$$JTC(n,T,t,K) = \left\{\left[nDT\left(\frac{DT}{R} + (n-1)T\right) - \frac{nDT\left(\frac{nDT}{R}\right)}{2}\right] - \left[nDT + \frac{D}{2}T^2 + \left(nD\sum_{k=0}^{\infty}\frac{a^k T^{\beta k+2}}{k!\,(\beta k+2)}\right) \times e^{-\alpha T^{\beta}}\right] \times \frac{hv}{nT}\right\} + \frac{Sv}{nT} + \frac{nF}{nT} + \frac{nULe^{-mK}}{nT} + \frac{D \times hb}{2} + Dp + \frac{D(T-t) \times p \times Ic}{2T} - \frac{D(t) \times p \times Id}{2T} + K$$

3.4.4 Forecasting the present value of joint total cost using continuous interest compounded Discounted Cash Flow (DCF) model.

The joint total cost function as per equation (17) is derived based on supplier-buyer agreement on the inventory planning of the supply chain purchase policy. DCF analysis is a very useful method to valuing a project using the concepts of the time value of money. The present value of joint total cost can be estimated by multiplying the projected joint total cost function (17) with designated inflation discount factor over time horizon. Discounted cash flow analysis is widely used in investment finance, real estate development and corporate financial management.

The logics behind applying inflation discount rate are: (1) the time value of money (the managers would make an effective economic decision by predicting EOQ, number of reorder quantity and the least total cost available in the projected procurement options) and (2) the risk premium that reflects the extra return of investor demand. Tariq and Melinda (2005) stated project cash flow often occur on a continuous basis rather at the end of each period. In this model, the projected joint total cost value take assumption of continuous integrated cash flow as a lump sum at the end of each period with accounting the continuous discounting factor. As per classification in (Tariq and Melinda, 2005) research, the joint projected costing analysis for this model is known as Discrete Cash Flows with Continuous Discounting.

In discrete DCF suggested the present value of amount V from nT period during H planning horizon (JTC (n,T,t,K) is assigned as V) can be generalized as:

$$PV_{JTC}=\frac{V}{(1+r)^n}\ ;\ r > 0 \text{ and } n > 0 \text{ (equation 18)}$$

where r is the inflation interest rate per period and compounding occurs at the end of each period. Note that the term n has open bound limit as $n \rightarrow +\infty$, so $PV_{JTC} \rightarrow 0$.

Equation (18) is designed to approximate a continuous discount frequency in a year. The discount factor will now consider n, number of interest compounded in a year. This assumption supports the statement above of continuous interest compounding on V. The DCF formula (equation 18) is now revised to:

$PV_{JTC}=\frac{V}{\left(1+\frac{r}{n}\right)^{n}}$; (equation 19) where n has open bound limit of $n\rightarrow +\infty$

Assessing only the equation (19) denominator with $n \rightarrow +\infty$, one has indeterminate form of 1^{∞}.L'Hopital rule suggest that the previous argument may follows one of the cases where:

$$\lim_{x\rightarrow a}\frac{f(x)}{g(x)}=\frac{0}{0} \text{ OR } \lim_{x\rightarrow a}\frac{f(x)}{g(x)}=\frac{\pm\infty}{\pm\infty}$$

where a can be any real number, ∞ or -∞. Using the common function of infinity of exponential as given by:

$$\lim_{n\rightarrow +\infty}\left(1+\frac{1}{s}\right)^{s}=e \text{ (equation 20)}$$

NOTE: Variable s is introduced for simplification.

In further reduction of equation (20) leads to introduction of new variable: $w=1/n$. Applying limit $n\rightarrow +\infty$ to w gives, $w\rightarrow 0$. by manipulating equation (20), one has:

$$\lim_{s\rightarrow +\infty}\left(1+\frac{1}{s}\right)^{s}=\lim_{w\rightarrow 0}(1+w)^{\frac{1}{w}}$$

Introduce a dependent variable of $y(w)$, one has:

$$y=(1+w)^{\frac{1}{w}}$$

Take the natural log at both side of equation above:

$$\ln y=\ln\left[(1+w)^{\frac{1}{w}}\right]=\frac{1}{w}\ln(1+w)$$

Thus,

$$\lim_{w\rightarrow 0}\ln y=\lim_{w\rightarrow 0}\left[\frac{\ln(1+w)}{w}\right]$$

Now, the equation above follows indeterminate form of type 0/0. Assess by L'Hopital rule, one has:

$$\lim_{w\to 0} \ln y = \lim_{w\to 0} \frac{\ln(1+w)}{w} = \lim_{w\to 0} \frac{1/(1+w)}{1} = 1$$

Thus, from the assessment, it justify that $\ln y \to 1$as $w \to 0$, as per equation (20), the continuity function yields $e^{\ln y} \to e^1$ as $w \to 0$ as shown that:

$$\lim_{n\to+\infty} \left(1+\tfrac{1}{s}\right)^{s} = \lim_{w\to 0}(1+w)^{\frac{1}{w}} = y = e \text{ (equation 21)}$$

Next step is to derive the continuous function of DCF. Taking the denominator of equation (19) to infinity, one has:

$$\lim_{n\to+\infty} \left(1+\frac{r}{n}\right)^{n}$$

Assigning a new variable $c = n/r$. Then $1/c = r/n$ and $n = cr$. Applying the same bound $n \to +\infty$, so $s \to +\infty$ and substitute into equation above yields:

$$\lim_{s\to+\infty} \left(1+\tfrac{1}{c}\right)^{(cr)T} = \lim_{s\to+\infty} \left[\left(1+\tfrac{1}{c}\right)^{c}\right]^{rT} \text{ (equation 22)}$$

Noting the function inside the bracket (22) equals to equation (21), by substitution of term in the bracket yields:

$$\lim_{s\to+\infty} \left[\left(1+\tfrac{1}{c}\right)^{c}\right]^{rT} = [e]^{rt} \text{ (equation 21)}$$

Substituting the equation (21) to the denominator of equation (19) yields the formula of discrete cash flow of present value of joint total cost with continuous discounting factor:

$$PV_{JTC} = \frac{V}{e^{rT}} = Ve^{-rT} \text{ (equation 22)}$$

where the term V is JTC (n,T,t,K).

Equation (22) is modified to suit into cumulative inventory model by adding sum series of n is given by:

$$PV_{JTC} = \frac{V}{e^{rnT}} = Ve^{-rnT} = Ve^{-rH} \text{ (equation 23)}$$

All time dependent inventory functions for supplier and buyer as derived above shall be subjected to continuous DCF with net discount rate of inflation, r.

3.4.5 **Joint quantity discount adjustment under demand uncertainty**

The quantity discount approach employed in the new model is similar to Benton (2007). To implement the quantity discount incentive, assumption (7) is relaxed by employing stochastic demand under stockout probability distribution rather than deterministic demand. Assume the quantity discount follows buyer EOQ and step size N as a strategic planning.

Let buyer annual total cost of equation (16) will be:

$$TC_b(n,T,t,K) = \frac{nF}{nT} + \frac{nULe^{-mK}}{nT} + \frac{D \times hb}{2} + Dp + \frac{D(T-t)\times p\times Ic}{2T} - \frac{D(t)\times p\times Id}{2T} + K$$

where $TC_b(n,T,t,K) = C1 + C2 + C3 + C4 + C5 - C6 + K$

Reducing (16) to single cycle cost function will give:

$$TC_b(n,T,t,K) = F + ULe^{-mK} + \frac{DT \times hb}{2} + DTp + \frac{D(T-t)^2\times p\times Ic}{2T} - \frac{D(t)^2\times p\times Id}{2T} + \frac{K}{n}$$

Let the price determined cost functions shall be modify into discount functions. C1 + C2 + K are grouped as X to simplify the derivation and gives:

$$TC_b(T,t) = X + \frac{DT \times hb}{2} + DTp + \frac{D(T-t)^2\times p\times Ic}{2T} - \frac{D(t)^2\times p\times Id}{2T} \text{ (equation 24)}$$

As per parameter definition in Table 3-1, permissible delay period, t = 80% T (equation 25) and EOQ(buyer),Q = DT. Substitute (25) and Q into (24) to reduce the equation regards with common term T:

$$TC_b(T) = X + \frac{Q \times hb}{2} + Qp + \frac{D(T-0.8T)^2\times p\times Ic}{2T} - \frac{D(0.8T)^2\times p\times Id}{2T}$$

Reduction of the total cost function is possible by assuming the holding cost is a linear function with price per unit as per below:

$p \times w = h_b$ (equation 26) where w is a constant.

Substituting the assigned parameter as per Table 3-1 where p = \$10 and h_b = \$5 into (26) gives w = 0.5. So, when $h_b = 0.5p$, the reduced function for buyer total cost before discount is:

$$TC_b(T) = X + 0.25Qp + Qp + 0.02Qp \times Ic - 0.32Qp \times Id \text{ (equation 27)}$$

Quantity discount, \$/unit is introduced as d_k. Modifying the (27) into discount total cost function with step size N gives:

$TC_b(N,T) = X + 0.25NQ(p - d_k) + NQ(p - d_k) + 0.02NQ(p - d_k)I_c - 0.32NQ(p - d_k)I_d$ (equation 28)

Assume the buyer purchase some safety stock to the inventory. The safety stock is defined as ss. The safety stock assumption is reasonable because buyer's safety stock can be purchased in advance before employing quantity discount policy. Safety stock cost function is given by $ss(h_b) \approx ss(h_b \times dk)$. Since in determining the quantity discount only involve price dependent cost function, the safety stock function is cancelled out in both equation.

To determine the quantity discount policy, the computation must satisfy the following constraint otherwise it is not make sense for buyer to accept the quantity discount.

$$TC_b(T) - TC_b(N,T) \geq 0 \text{ @ Total cost (before – after discount)} \geq 0$$

$$[X + 0.25Qp + Qp + 0.02Qp \times Ic - 0.32Qp \times Id + ss(h_b)] - [X + 0.25NQ(p - d_k) + NQ(p - d_k) + 0.02NQ(p - d_k)I_c - 0.32NQ(p - d_k)I_d + ss(h_b)] \geq 0 \text{ (equation 29)}$$

Arranging the cost function by setting the constraint equals to zero, the quantity discount schedule is given by:

$$d_k = p\left[\frac{(N-1)(1.25Q+0.02Q\times Ic-0.32Q\times Id)}{NQ(1.25+0.02Ic-0.32Id)}\right] \text{ (equation 30)}$$

But increase the order quantity cause the buyer to overstock which shall lead to increment of holding cost over the planning horizon. So, by using buyer risk adjustment approach which similar to Benton (2007), the overstock cost can be determined.

Stochastic demand distribution is used to estimate actual demand normal distribution as an independent random variable. Buyer mean actual demand per cycle is defined as G and overstock condition as O. Overstock condition is defined as per constraint below:

$$O = \begin{cases} NQ - G & ; if\ G < NQ \\ 0 & ; if\ G \geq NQ \end{cases}$$

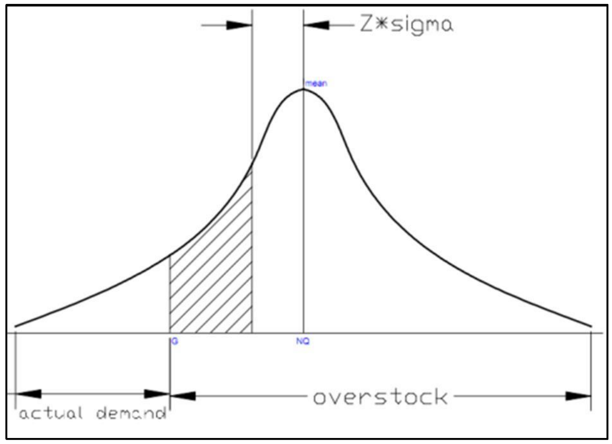


Figure 3-8 Benton (2007) Normal distribution of overstocking probability with supplier risk sharing at Zσ

The overstock condition tells if actual demand is less than estimated demand, overstock will occur and vice versa. Supplier is relaxed by sharing the overstock cost if the actual demand is less than order quantity in acceptable (marginal) manner. Supplier need to determine certain proportion of overstock which can be shared by supplier. As per Benton (2007), a decision variable Z is introduced to define accepted overstock proportion. At Z = 0, the supplier shall not tolerate overstock cost sharing which is carried by the buyer. Modified the overstock condition based on Figure 3-8 Benton (2007) Normal distribution of overstocking probability with supplier risk sharing at Zσ with variable Z as cutoff point is given by:

$$O^Z = \begin{cases} (NQ - Z\sigma) - G & ; if\ G < (NQ - Z\sigma) \\ 0 & ; if\ G \geq (NQ - Z\sigma) \end{cases}$$

Estimated overstock function is given by:

$$O^Z = \int_{-\infty}^{KQ - Z\sigma} [(NQ - Z\sigma) - G]\ f(G).d(G) \text{ (equation 31)}$$

Equation (31) can be modified by integrating standard score, u = $\frac{G-KQ}{\sigma} \sim N(0,1)$. Employing normal distribution assumption yields:

$$O^Z(u) = \sigma \int_{-\infty}^{-Z} (-Z - u)\ f(u).d(u) \text{ (equation 32)}$$

A new term substitution to the normal distribution function (32) is given by

$$E(u) = \int_{-\infty}^{-Z} (-Z - u)\, f(u).\, d(u) \text{ (equation 33)}$$

Thus, by substituting (33) into (32) and assigning the cost factor into the function yields the overstocking cost per annual term:

$$O^Z(u) = \frac{1}{2}\big(\sigma E(u)\big)\big(0.8(p - dk)\big) = 0.4\sigma E(u) \times (p - d_k) \text{ (equation 34)}$$

The value of E(u) is depends on Z, the acceptable quantity for buyer to share the overstock cost carried by the buyer.

Bringing back (29) by incorporating overstock cost before and after quantity discount yield a new quantity discount schedule for buyer:

$[X + 0.25Qp + Qp + 0.02Qp \times Ic - 0.32Qp \times Id + ss(h_b) + 0.4\sigma E(u) \times p] - [X + 0.25NQ(p - d_k) + NQ(p - d_k) + 0.02NQ(p - d_k)I_c - 0.32NQ(p - d_k)I_d + ss(h_b) + 0.4\sigma E(u) \times (p - d_k)] \geq 0$ (equation 35)

Arranging the cost function by setting the constraint equals to zero, the new quantity discount schedule is given by:

$$d_k = p\left[\frac{(N-1)(1.25Q + 0.02Q \times Ic - 0.32Q \times Id) + 0.4\sigma E(u)}{NQ(1.25 + 0.02Ic - 0.32Id) + 0.4\sigma E(u)}\right] \text{ (equation 36)}$$

3.5 Mathematical Model

The analysis without some parameters defined is too complex to be done. Therefore, we limit into few factors that are significant in the decision policy. The Table 3-1below shows the parameters that are earlier defined to simplify the derivation.

In determine how much deterioration occurs between two successive shipping period, T, assume the supplier is employing 3 sigma of in its inventory quality management. To be realistic, number of deterioration is always follows non-integer number which can lead to complex gamma function. Although it is not realistic by allowing deterioration follows a discrete number over time but presuming the quality 3 sigma approach is more preferable in this model. Therefore, by allowing 3 sigma in each shipment the buyer will have 2.7 defective products out of 1000 parts. It is about 0.26998% of parts deteriorated in each shipment. Since the model only allows discrete number of deterioration per T, thus 2.7 defective products is rounded to 3. Rounding the value to the next integer shall treat as a safety factor in designing the deterioration model.

Parameter	Value
Number of deterioration, n	3
Annual demand, D (unit/year)	3000
Production rate, R (unit/year)	6500
Deterioration scale factor, α	1
Deterioration shape factor, β	1
Net discount of inflation rate, r ($/$1/year)	0.10
Purchase price, P($/unit)	10
Interest charged, I_c($/$1/year)	0.15
Interest earned,I_e($/$1/year)	0.12
Supplier holding cost, h_v($/unit/year)	1.00
Buyer holding cost, h_b($/unit/year)	5.00
Supplier setup cost, S_v($/production run)	200.00
Fixed transportation cost per shipment, F ($/shipment)	300.00
Buyer order processing cost, U ($/order per time)	560.00
Time for order processing per shipment, L (order time, year)	0.15
Capital investment for employing the order processing strategy, K ($/planning horizon)	80
Exponential smoothing constant, m	0.008
Safety stock, ss (unit)	300
Standard deviation, σ (unit)	as per Table 4-3, Table 4-6 and Table 4-9
Supplier's risk sharing proportion, Z (%)	30%, 50%, 80%

Table 3-1 Assigned Parameter

3.5.1 **Present value of supplier inventory model**

The series expansion of deterioration factor in equation (10) is expanded to $k \to 3$ and let α and β = 1:

$$\sum_{k=0}^{\infty} \frac{a^k T^{\beta k+2}}{k!\,(\beta k+2)} = \sum_{k=0}^{3} \frac{(1)^k T^{(1)k+2}}{k!\,((1)k+2)} = \left[\frac{T^2}{2} + \frac{T^3}{3} + \frac{T^4}{2!\,(4)} + \frac{T^5}{3!\,(5)}\right]$$

substitute the terms in bracket above into cumulative depleting inventory (10) is given by:

$I(n,T) = nDT + \frac{D}{2}T^2 + \left(nD\left[\frac{T^2}{2} + \frac{T^3}{3} + \frac{T^4}{2!(4)} + \frac{T^5}{3!(5)}\right]\right) \times e^{-T}$ (equation 37)

Bringing back the supplier inventory model here with correction from (37):

I(n,T) = $\left\{\left[nDT\left(\frac{DT}{R} + (n-1)T\right) - \frac{nDT\left(\frac{nDT}{R}\right)}{2}\right] - \left(nDT + \frac{D}{2}T^2 + \left(nD\left[\frac{T^2}{2} + \frac{T^3}{3} + \frac{T^4}{2!(4)} + \frac{T^5}{3!(5)}\right]\right) \times e^{-T}\right)\right\}$ (equation 38)

As suggested by Tripathi (2010), the first order cycle is subjected to finite integration with DCF. Then the total cost is easily by multiple the exponential projections over the time horizon H general equation. But (38) is in cumulative function over the planning horizon. By simply divide with (38), one gets the <u>average inventory at single cycle period</u> as given below:

I(n,T) = $\left\{\left[DT\left(\frac{DT}{R} + (n-1)T\right) - \frac{DT\left(\frac{nDT}{R}\right)}{2}\right] - \left(DT + \frac{D}{2n}T^2 + \left(D\left[\frac{T^2}{2} + \frac{T^3}{3} + \frac{T^4}{2!(4)} + \frac{T^5}{3!(5)}\right]\right) \times e^{-T}\right)\right\}$ (equation 39)

Firstly, integrate the cumulative production inventory model on the left side (39) with the discount factor with closed domain of (0,T) at specific number of replenishment, n.

$$\int_0^T DT\left(\frac{D}{R}\right)Te^{-rT}dT + \int_0^T DT(n-1)Te^{-rT}dT - \int_0^T \frac{1}{2R}DT(nDT)e^{-rT}dT$$

$$=\frac{D^2}{R}\int_0^T T^2e^{-rT}dT + D(n-1)\int_0^T T^2e^{-rT}dT - \frac{nD^2}{2R}\int_0^H T^2e^{-rT}dT$$

$$=\left(-\frac{e^{-rT}}{r^3}[r^2T^2+2rT+2]\Big|_0^T\right)\times\left(\frac{D^2}{R}+D(n-1)-\frac{nD^2}{2R}\right) \text{ (equation 40)}$$

and similar to the cumulative inventory depletion (21):

$$\int_0^T\left(\left[DT+\frac{D}{2n}T^2+\left(D\left[\frac{T^2}{2}+\frac{T^3}{3}+\frac{T^4}{2!\,(4)}+\frac{T^5}{3!\,(5)}\right]\right)\times e^{-T}\right]e^{-rT}\right)dT$$

For the inventory depletion model, left hand side (inventory depletion due to demand rate) =

$$D\left(-\frac{e^{-rT}}{r^2}[rT+1]\Big|_0^T\right)+\frac{D}{2n}\left(-\frac{e^{-rT}}{r^3}[r^2T^2+2rT+2]\Big|_0^T\right) \text{ (equation 41)}$$

Right hand side of inventory depletion due to deterioration rate =

$$=D\left(\frac{1}{2}\int_0^T T^2e^{-T(r+1)}dT+\frac{1}{3}\int_0^T T^3e^{-T(r+1)}dT+\frac{1}{2!(4)}\int_0^T T^4e^{-T(r+1)}dT+\frac{1}{3!(5)}\int_0^T T^5e^{-T(r+1)}dT\right)$$

$$=\Big[\frac{D}{2(r+1)^3}\left(e^{-T(r+1)}\right)(-T^2(r+1)^2-2T(r+1)-2)+\frac{D}{3(r+1)^4}\left(e^{-T(r+1)}\right)(-T^3(r+1)^3-3T^2(r+1)^2-6T(r+1)-6)+\frac{D}{2!(4)(r+1)^5}\left(e^{-T(r+1)}\right)(-T^4(r+1)^4-4T^3(r+1)^3-12T^2(r+1)^2-24T(r+1)-24)+\frac{D}{3!(5)(r+1)^6}\left(e^{-T(r+1)}\right)(-T^5(r+5)^5-5T^4(r+1)^4-20T^3(r+1)^3-60T^2(r+1)^2-120T(r+1)-120)\Big]\Big|_0^T \text{(equation 42)}$$

*NOTE:

- The discount rate of inflation is defined by r = $ cash value portion per $ 1 dollar per year.

Due to tedious equation presented above, it is wise to reduce the function to more convenience approach by fixing the discount rate of inflation, *r =$0.10/$/year.*

Using the assumption of the average inventory of single cycle period is equal to first replenishment inventory cycle, the present value for supplier first inventory cycle based on r = 0.10 is given by:

$$I(n,T) = \left(-\frac{e^{-0.1T}}{0.01}[0.01T^2 + 0.2T + 2] + \frac{1}{0.01}(2)\right) \times \left(\frac{D^2}{R} + D(n-1) - \frac{nD^2}{2R}\right) - \left\{\left(D\left[-\frac{e^{-0.1T}}{0.01}[0.1T + 1] + \frac{1}{0.01}(1)\right] + \frac{D}{2n}\left[-\frac{e^{-0.1T}}{0.001}[0.01T^2 + 0.2T + 2] + \frac{1}{0.001}(2)\right]\right) + \left[\left(\frac{D}{2.662}(e^{-1.1T})(-T^2(1.21) - T(2.2) - 2)\right) + \left(\frac{D}{4.3923}(e^{-1.1T})(-T^3(1.331) - T^2(3.63) - T(6.6) - 6)\right) + \left(\frac{D}{12.884}(e^{-1.1T})(-T^4(1.4641) - T^3(5.324) - T^2(14.52) - T(26.4) - 24)\right) + \left(\frac{D}{53.148}(e^{-1.1T})(-T^5(1.6105) - T^4(7.3205) - T^3(26.62) - T^2(72.6) - T(132) - 120)\right)\right]\right\} \quad \text{(equation 43)}$$

3.5.2 **Present value of total cost for supplier inventory planning model**

In the initial attempt to incorporate present value projection into each individual function in (38) would be difficult to be done. By using the steps as per Tripathi (2010), the present value of total cost for supplier can be easily being employed as per:

$$\mathrm{PVTC_b(n,T)} = \mathrm{h_b} \times \sum_{j=0}^{n-1} e^{-jrT} \times PV(I(T)) + \mathrm{s_v} \times \sum_{j=0}^{n-1} e^{-jrT}$$

$$\mathrm{PVTC_b(n,T)} = \mathrm{h_b} \times \frac{1-e^{-jrT}}{1-e^{-rT}} \times \left(\left(-\frac{e^{-0.1T}}{0.01}[0.01T^2 + 0.2T + 2] + \frac{1}{0.01}(2)\right) \times \left(\frac{D^2}{R} + D(n-1) - \frac{nD^2}{2R}\right) - \left\{\left(D\left[-\frac{e^{-0.1T}}{0.01}[0.1T + 1] + \frac{1}{0.01}(1)\right] + \frac{D}{2n}\left[-\frac{e^{-0.1T}}{0.001}[0.01T^2 + 0.2T + 2] + \frac{1}{0.001}(2)\right]\right) + \left[\left(\frac{D}{2.662}(e^{-1.1T})(-T^2(1.21) - T(2.2) - 2)\right) + \left(\frac{D}{4.3923}(e^{-1.1T})(-T^3(1.331) - T^2(3.63) - T(6.6) - 6)\right) + \left(\frac{D}{12.884}(e^{-1.1T})(-T^4(1.4641) - T^3(5.324) - T^2(14.52) - T(26.4) - 24)\right) + \left(\frac{D}{53.148}(e^{-1.1T})(-T^5(1.6105) - T^4(7.3205) -\right.\right.\right.$$

$T^3(26.62) - T^2(72.6) - T(132) - 120)\Big)\Big]\Big\}\Bigg)$ $+s_v \times \left(\frac{1-e^{-jrT}}{1-e^{-rT}}\right)$ (equation 39)

3.5.3 **Present value of total cost for buyer's inventory planning model**

To provide the present value projection over time horizon H, each of individual of the demand dependent inventory function is integrated with net discount of inflation rate factor with respect to replenishment cycle period, T. The domain for the integral functions is a closed domain at (0,T).

$PVTC_b(n,T,t,K)=\int_0^T \left(\frac{nF}{nT} + \frac{nULe^{-mK}}{nT} + \frac{D \times hb}{2} + Dp + \frac{D(T-t)\times p\times Ic}{2T} - \frac{D(t)\times p\times Id}{2T} + K\right)(e^{-rT})dT$ (equation 40 – annual planning)

$PVTC_b(n,T,t,K)=\int_0^T \left(nF + nULe^{-mK} + \frac{nDT \times hb}{2} + nDTp + \frac{nD(T-t)(T-t)\times p\times Ic}{2T} - \frac{nDt(t)\times p\times Id}{2T} + nTK\right)(e^{-rT})dT$ (equation 41 – global purchase planning over the time horizon)

where $PVTC_b(n,T,t,K) = \int_0^T (C1 + C2 + C3 + C4 + C5 - C6 + K)\,(e^{-rT})dT$

Equation (40) is in form of annual value function and (41) is the global purchase policy over the time horizon for buyer. To implement the DCF approach in predicting the present value of total cost, Cost function of (41) has to be reduced into single cycle period to project the value over the purchase planning horizon by simply dividing each function with number of replenishment, n.

$PVTC_b(n,T,t,K)=\left(F + ULe^{-mK} + \frac{DT \times hb}{2} + DTp + \frac{D(T-t)(T-t)\times p\times Ic}{2T} - \frac{Dt(t)\times p\times Id}{2T} + K\right)\left(\sum_{j=0}^{n-1} e^{-rT}\right)$ (equation 42)

Evaluation for each of cost function (42) with regards net discount of inflation rate is given by:

C1: Fixed transportation cost = $F \times \sum_{j=0}^{n-1} e^{-jrT} = F \times \left(\frac{1-e^{-jrT}}{1-e^{-rT}}\right)$

C2: Order processing cost = $ULe^{-rK} \times \sum_{j=0}^{n-1} e^{-jrT} = ULe^{-mK} \times \left(\frac{1-e^{-jrT}}{1-e^{-rT}}\right)$

C3: Buyer holding cost = $h_b \times \frac{DT}{2} \times \sum_{j=0}^{n-1} e^{-jrT}$

$= h_b \times \frac{DT}{2} \times \left(\frac{1-e^{-jrT}}{1-e^{-rT}}\right)$

C4: Purchase cost = $DTp \times \sum_{j=0}^{n-1} e^{-jrT} = DTp \times \left(\frac{1-e^{-jrT}}{1-e^{-rT}}\right)$– cost estimation from initial inventory period

C5: Interest charged = $p \times I_c \times \frac{D(T-t)^2}{2T} \times \sum_{j=0}^{n-1} e^{-jrT}$

$= p \times I_c \times \frac{D(T-t)^2}{2T} \times \left(\frac{1-e^{-jrT}}{1-e^{-rT}}\right)$

C6: Interest earned = $p \times I_d \times \frac{D(t)^2}{2T} \times \sum_{j=0}^{n-1} e^{-jrT}$

$= p \times I_d \times \frac{D(t)^2}{2T} \times \left(\frac{1-e^{-jrT}}{1-e^{-rT}}\right)$

K: Capital investment per cycle period to operate the planned ordering system

$= T \times K \times \sum_{j=0}^{n-1} e^{-jrT} = T \times K \times \left(\frac{1-e^{-jrT}}{1-e^{-rT}}\right)$

3.5.4 **Present value of supplier and buyer joint total cost regard to net discount rate of inflation**

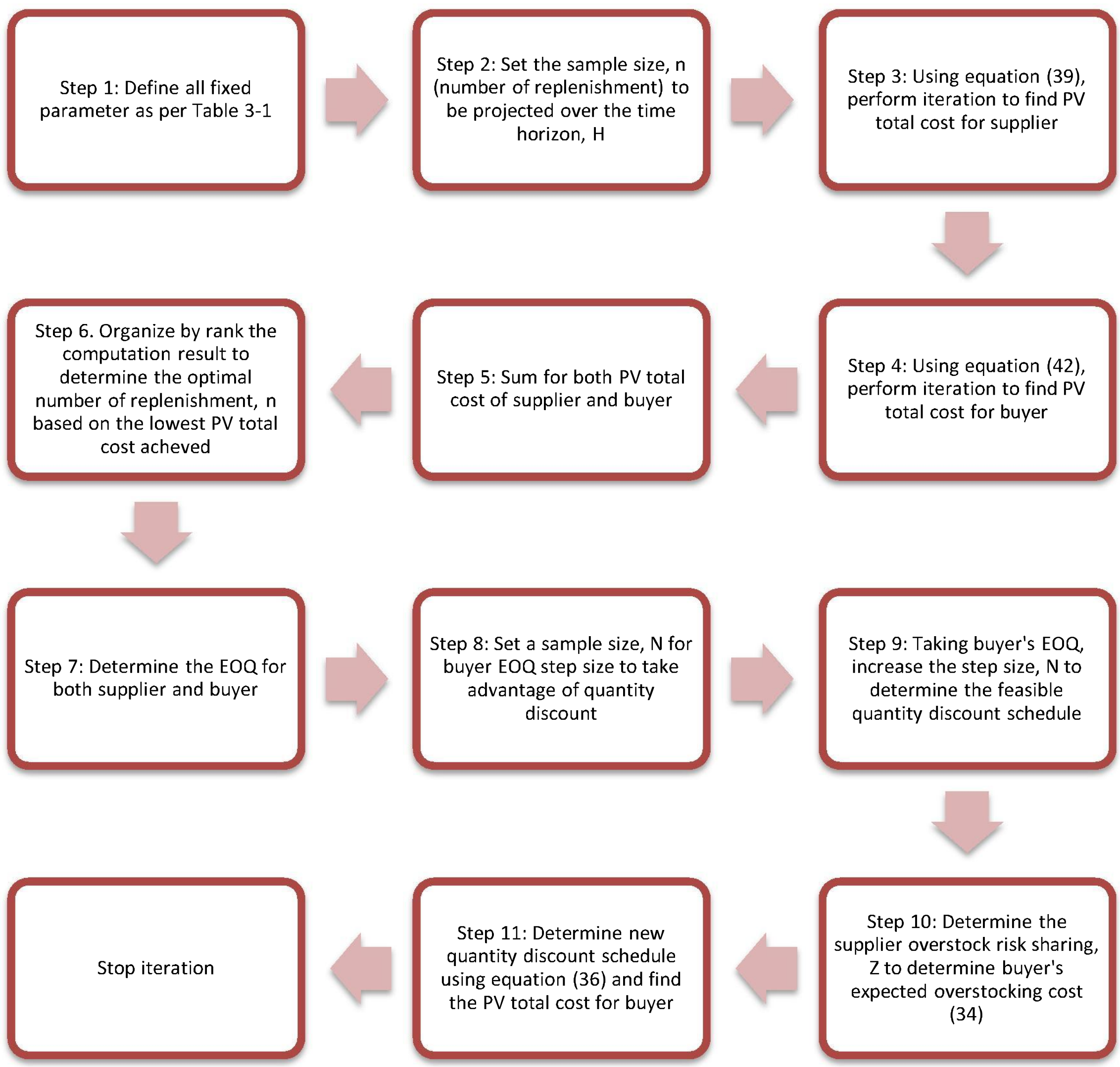
Thus, the present value of joint total cost model is given by:

PVJTC(n,T,t,K) = $PVTC_b$(n,T) + $PVTC_b$(n,T,t,K)

$$PVJTC(n,T,t,K) = h_b \times \left(\frac{1-e^{-jrT}}{1-e^{-rT}}\right) \times \left(\left(-\frac{e^{-0\ .1T}}{0.01}[0.01T^2 + 0.2T + 2] + \frac{1}{0.01}(2)\right) \times \left(\frac{D^2}{R} + D(n-1) - \frac{nD^2}{2R}\right) - \left\{\left(D\left[-\frac{e^{-0\ .1T}}{0.01}[0.1T + 1] + \frac{1}{0.01}(1)\right] + \frac{D}{2n}\left[-\frac{e^{-0\ .1T}}{0.001}[0.01T^2 + 0.2T + 2] + \frac{1}{0.001}(2)\right]\right) + \left[\left(\frac{D}{2.6\ 6\ 2}(e^{-1.1T})(-T^2(1.21) - T(2.2) - 2)\right) + \left(\frac{D}{4.3923}(e^{-1.1T})(-T^3(1.331) - T^2(3.63) - T(6.6) - 6)\right) + \left(\frac{D}{12.884}(e^{-1.1T})(-T^4(1.4641) - T^3(5.324) - T^2(14.52) - T(26.4) - 24)\right) + \left(\frac{D}{53.148}(e^{-1.1T})(-T^5(1.6105) - T^4(7.3205) - T^3(26.62) - T^2(72.6) - T(132) - 120)\right)\right]\right\}\right) + s_v \times \left(\frac{1-e^{-jrT}}{1-e^{-rT}}\right) + \left(F + ULe^{-mK} + \frac{DT \times hb}{2} + DTp + \frac{D(T-t)(T-t)\times p \times Ic}{2T} - \frac{Dt(t)\times p \times Id}{2T} + K\right)\left(\frac{1-e^{-jrT}}{1-e^{-rT}}\right) \text{ (equation 43)}$$

3.6 Mathematical Algorithm

Below is the proposed operation step in achieving quantity optimization and feasible total cost strategy for the new model. The computation is done by using software such as Microsoft Math, Microsoft Excel and Wolfram Mathematica. The spreadsheets of calculation are presented in the appendix



4 CHAPTER 4: RESULTS AND DISCUSSION

4.1 Computational Example

There are two economic factors considered in this model which are: (1) net discount of inflation rate and (2) order processing lead time reduction in buyer point of view. In the purchase order policy also offers two types of incentives for the buyer which are: (1) cash discount benefit from trade credit policy and (2) quantity discount incentive to take advantage of benefit in long run of business.

The objective is to determine the optimal joint economic order quantity driven by optimal number of replenishment which leads to optimal (minimum) joint total cost for supplier and buyer for mutual benefit over the planning horizon.

4.1.1 Present value cost analysis for supplier

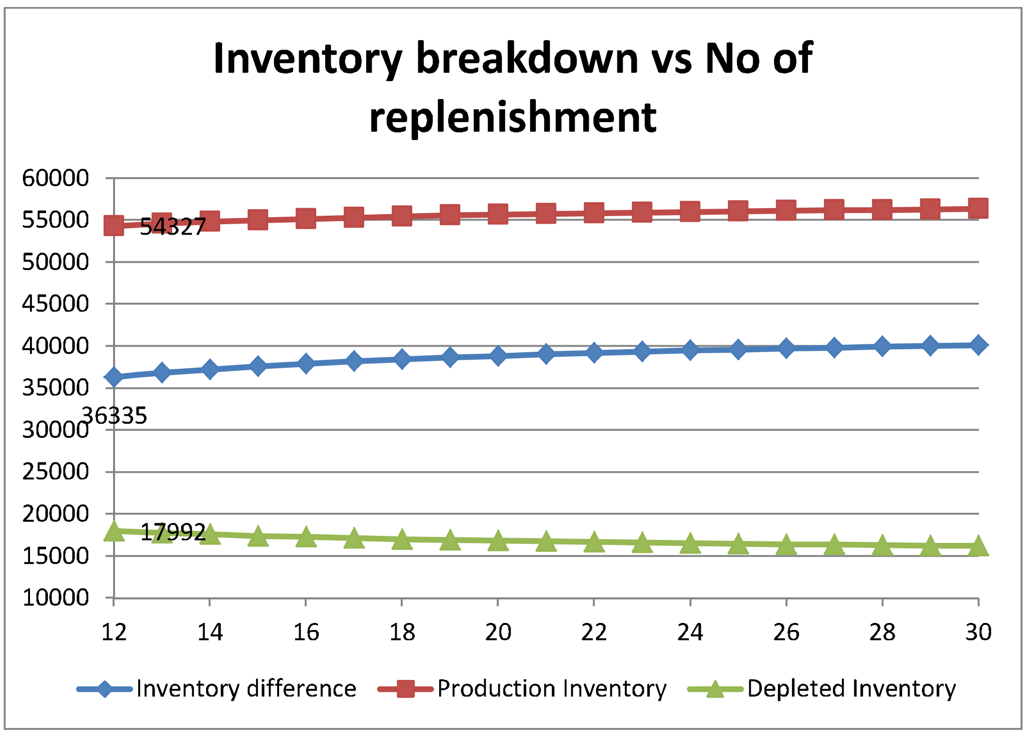


Figure 4-1 Inventory breakdown for supplier

Holding cost is often guided by the average inventory hold by supplier during the inventory cycle period over time horizon. Figure 4-1is projected based on Figure

3-5 which indicates the overall inventory changes over the planning horizon. As per Figure 3-1, the proposed supplier inventory cycle suggest that supplier implement a leading strategy in periodic increments where supplier increase its' production capacity even exceeding the current demand in the beginning to compensate the requirement from buyer over the planning horizon. Thus, this justifies the Figure 4-1as per increasing of cumulative inventory trend over the number of replenishment, n.

The management could also add enough capacity in one period to handle expected demand for multiple periods. This shall be discussed in the later section of quantity discount incentive.

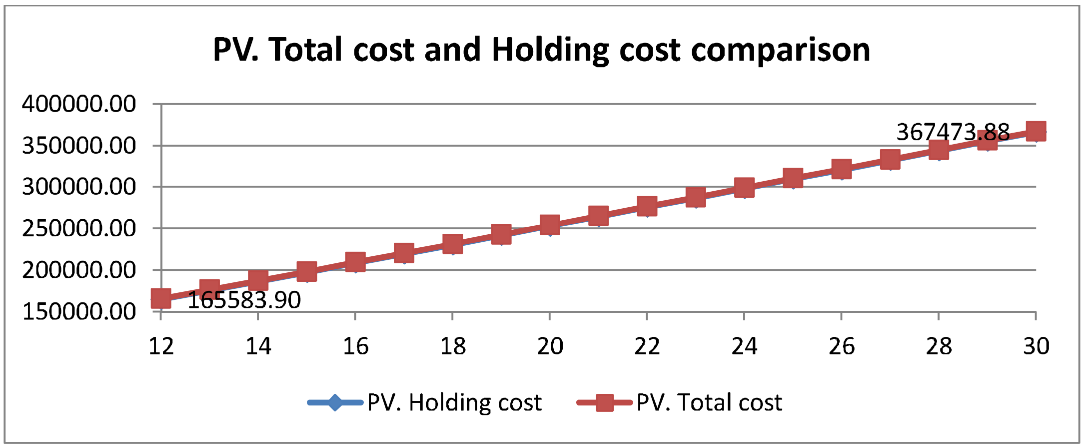


Figure 4-2 Present value cost analysis for H = 5 years (Due to major cost contribution, holding cost overlapped with the total cost)

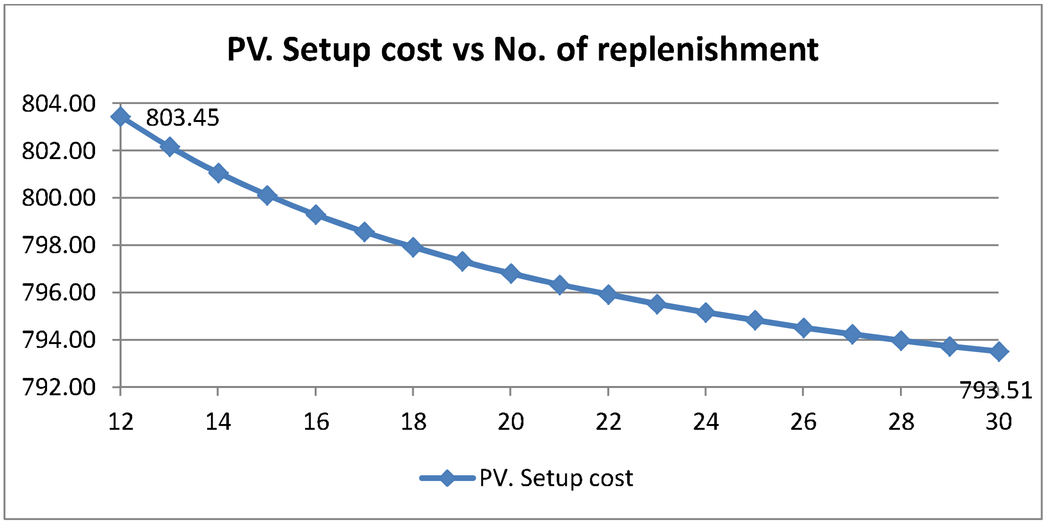


Figure 4-3 Supplier minor cost analysis: Setup cost

Due to large percentage of cost is biased towards supplier holding cost; the cost analysis diagram is presented in separate manner. The costing analysis as projected in Figure 4-1 also indicates the major cost contribution for supplier is the holding cost whereas the minor cost contribution is setup cost. As per Figure 4-3, the setup cost is decreasing with number of replenishment. Although in this model it is the minor cost breakdown, setup cost can also be a significant contribution in total cost estimation when a case requires a high complexity in expertise, skills, knowledge in preparation such as petrochemical process plant which yield a larger cost weightage/criticality rating over the total cost.

4.1.2 **Present value cost analysis for buyer (manufacturer)**

Estimation of buyer cost breakdown with effect of inflation rate is done by taking the average cost value over the sample size, which is number of replenishment, n. Detail iteration for cost breakdown is presented in the appendix.

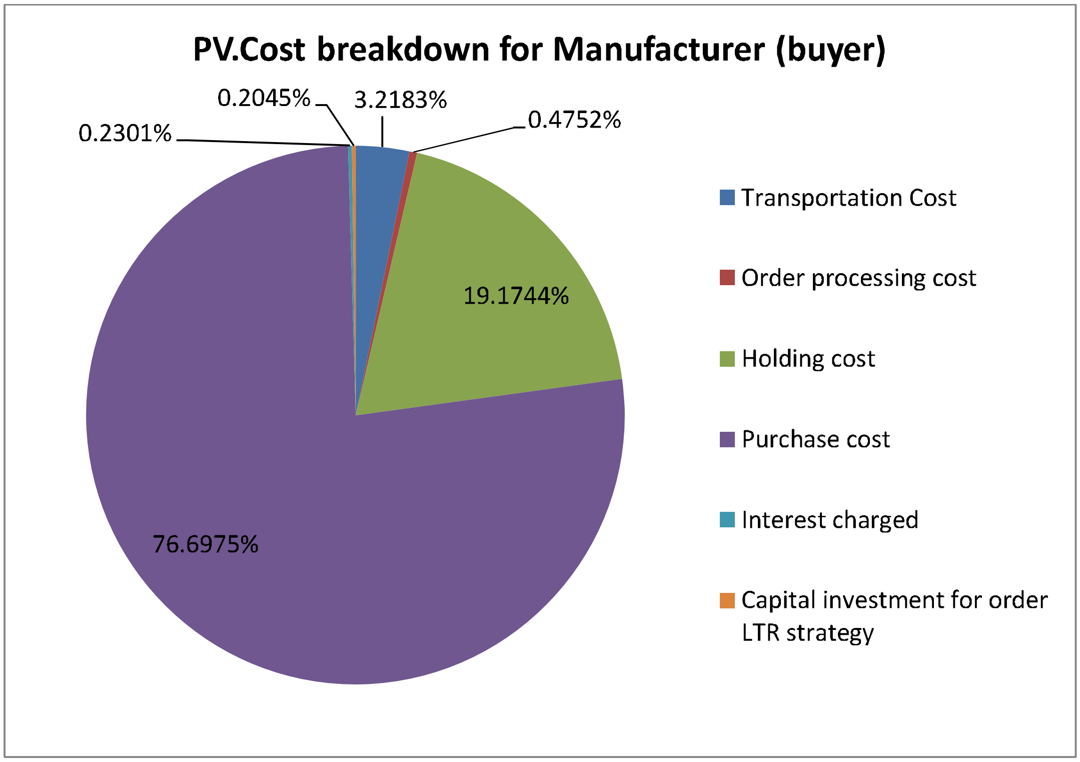


Figure 4-4 Supplier cost contribution breakdown

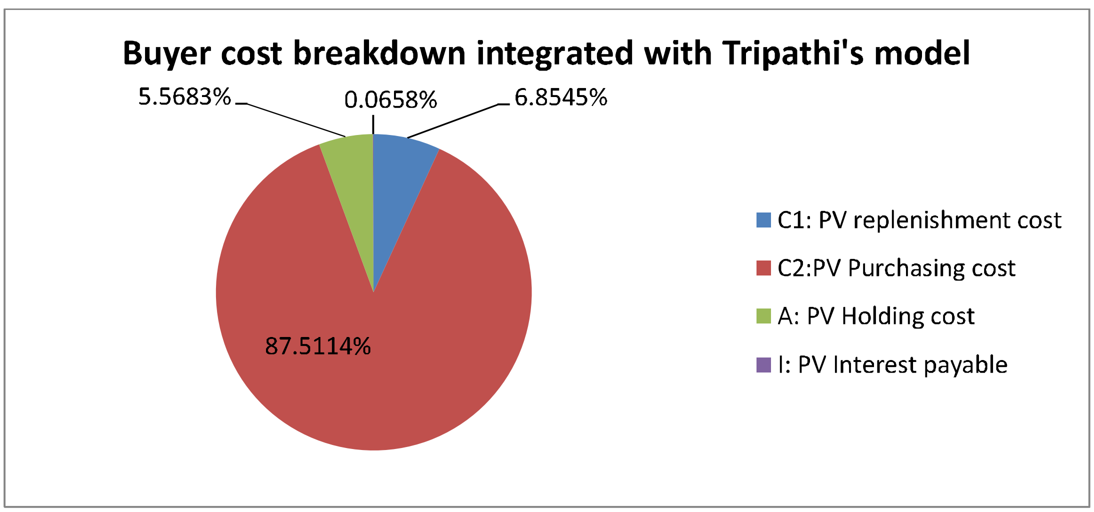


Figure 4-5 Buyer cost breakdown integrated with Tripathi's model: EOQ cash flow optimization for non-deteriorating item

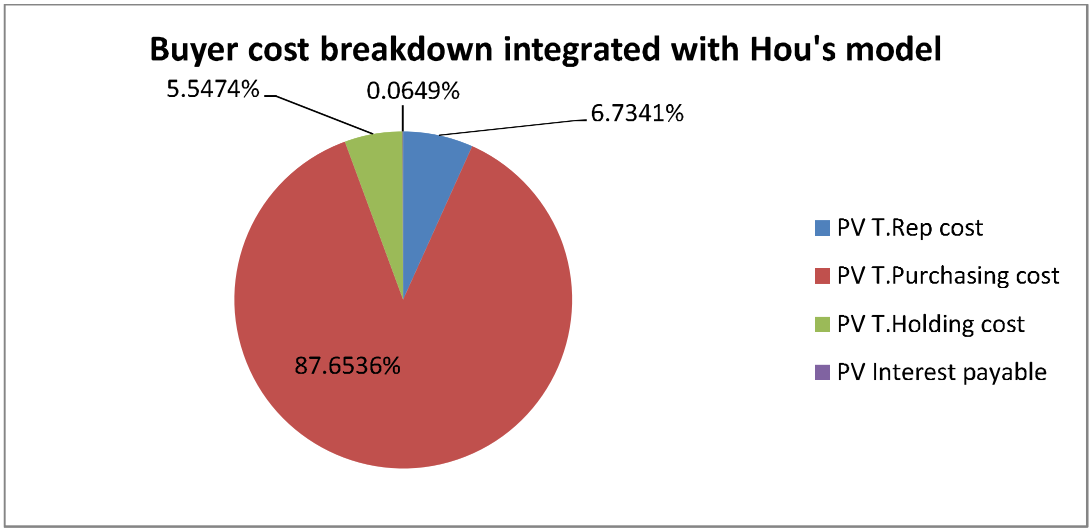


Figure 4-6 Buyer cost breakdown integrated with Hou's model: EOQ cash flow optimization for deteriorating item

Figure 4-4 indicates the largest cost contributor is purchase cost by 77% and second by holding cost by 19% and classifies as two major costs in the buyer inventory cycle. Whereas, insignificant contributor or minor costs comprise of transportation cost, order processing cost, interest charged and investment in order decision costs.

Using the same parameter for Tripathi (2010) and Hou (2009) as per Figure 4-5 and Figure 4-6, this model shows a common major cost contributor which is the purchasing cost. It is because the inflation factor incorporate in this model forecast the devaluation of money in the future that shall force a company to invest more capital in purchasing than present by rate factor of \$0.10 per \$ 1 per year over the time horizon, H = 5 years.

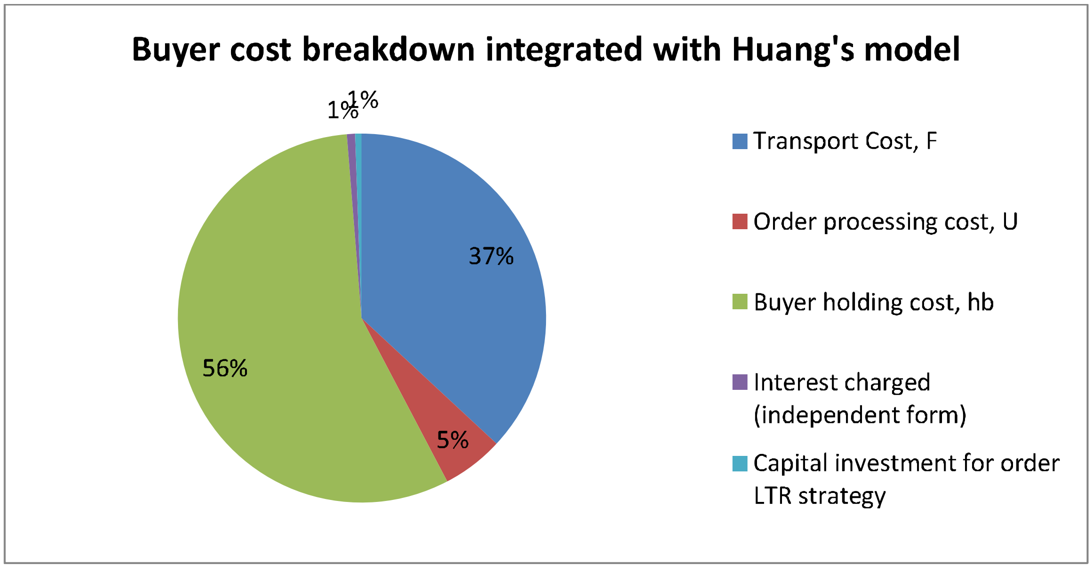


Figure 4-7 Buyer cost breakdown without inflation factor integrated with Huang's model.

This model uses the same buyer inventory cycle and cost breakdown as suggested by Huang (2010) as presented in Figure 4-7. In ideal estimation, holding cost should be the major cost contributor for the model. The transportation cost is the second larger cost contributor. However Huang does not consider purchase cost as suggested by Tripathi (2010). In conclusion, both cost breakdown projection before and after inflation is essential to determine the critical driving cost in the purchase decision strategy. Combined the critical costs determined from the cost breakdown analysis are <u>purchasing cost</u>, <u>holding cost</u> and <u>transportation cost</u>.

4.1.2.1 Detail projection of buyer major and minor cost for the new model.

Due to distinctive cost range, the buyer cost breakdown chart is presented separately as per below.

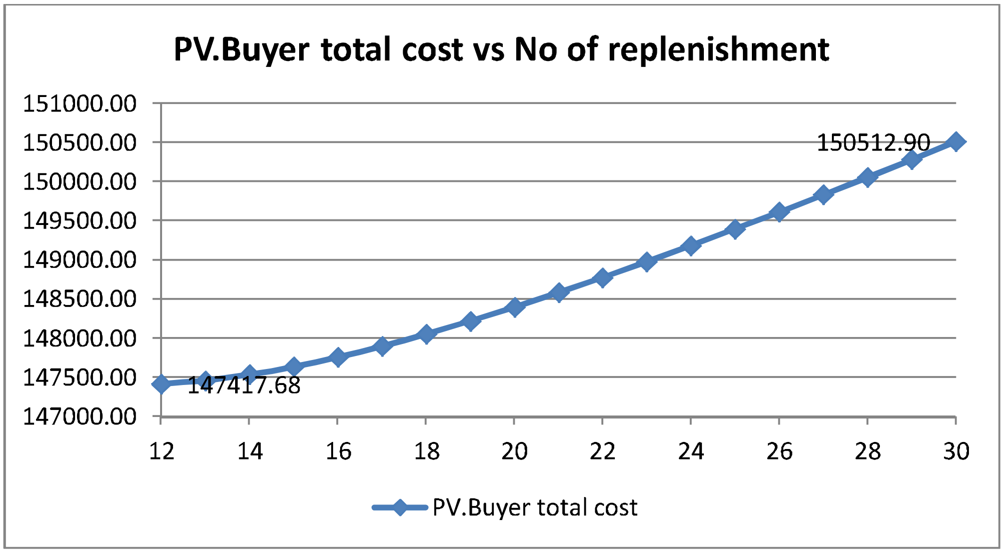


Figure 4-8 Buyer projected present value total cost

Figure 4-8 indicates the optimal buyer total cost projection is at n = 12 for $ 147,417.68. DCF with inflation suggest the buyer total cost shall increase with number of replenishment.

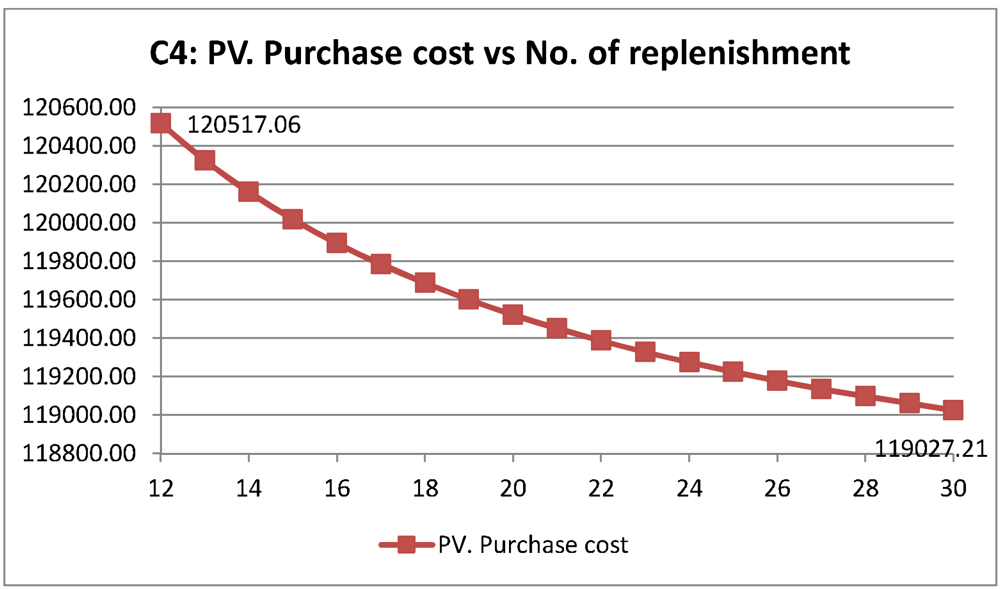


Figure 4-9 C4: Major cost – Present value of purchasing cost

As shown in Figure 4-4, C4: Purchase cost is the largest portion in the costing analysis. However, Figure 4-9 suggest as number of replenishment increase, product of purchase cost will decrease and stabilize at higher number of replenishment. This is because the cost function factor of cycle time inversely proportional to the number of replenishment.

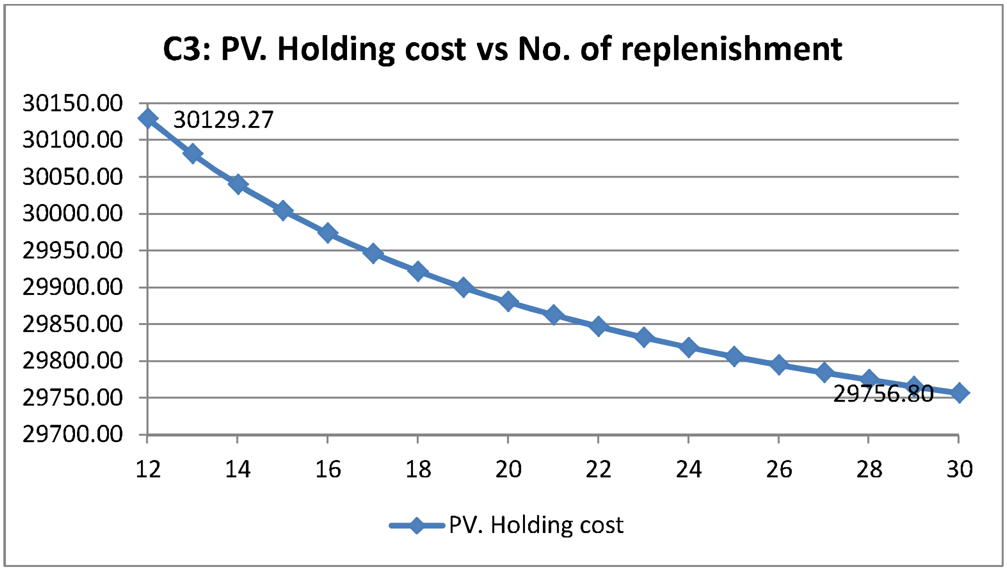


Figure 4-10 C3: Major cost – Present Value of holding cost

Second largest cost is holding cost. Holding cost function is a product of inventory dynamic flow with holding cost per unit per year. Holding cost is valued in currency $ and subjected to inflation which causes the money devalued continuously with discount rate of r. In this model, some parameter is kept constant such as planning horizon, H and inflation rate, r to simplify the mathematical model (30). Strategic planning suggests that n = 30 yields the lowest holding cost. However, the decision on number of replenishment has to be considered by at looking by global cost factor which are the major costs contributor.

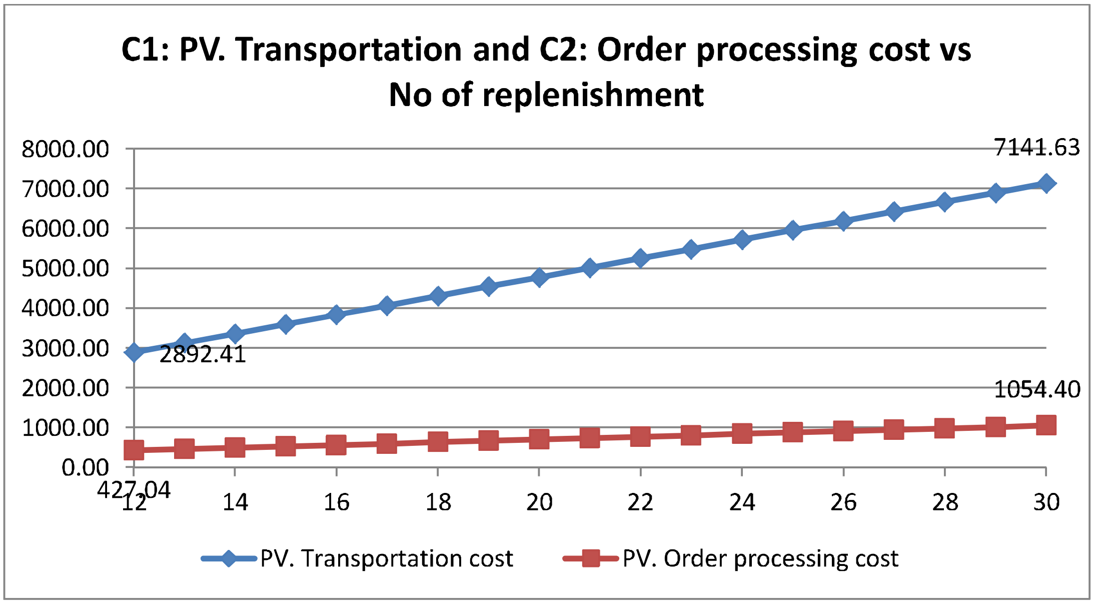


Figure 4-11 C1 and C2: Minor cost: PV transportation and order processing cost

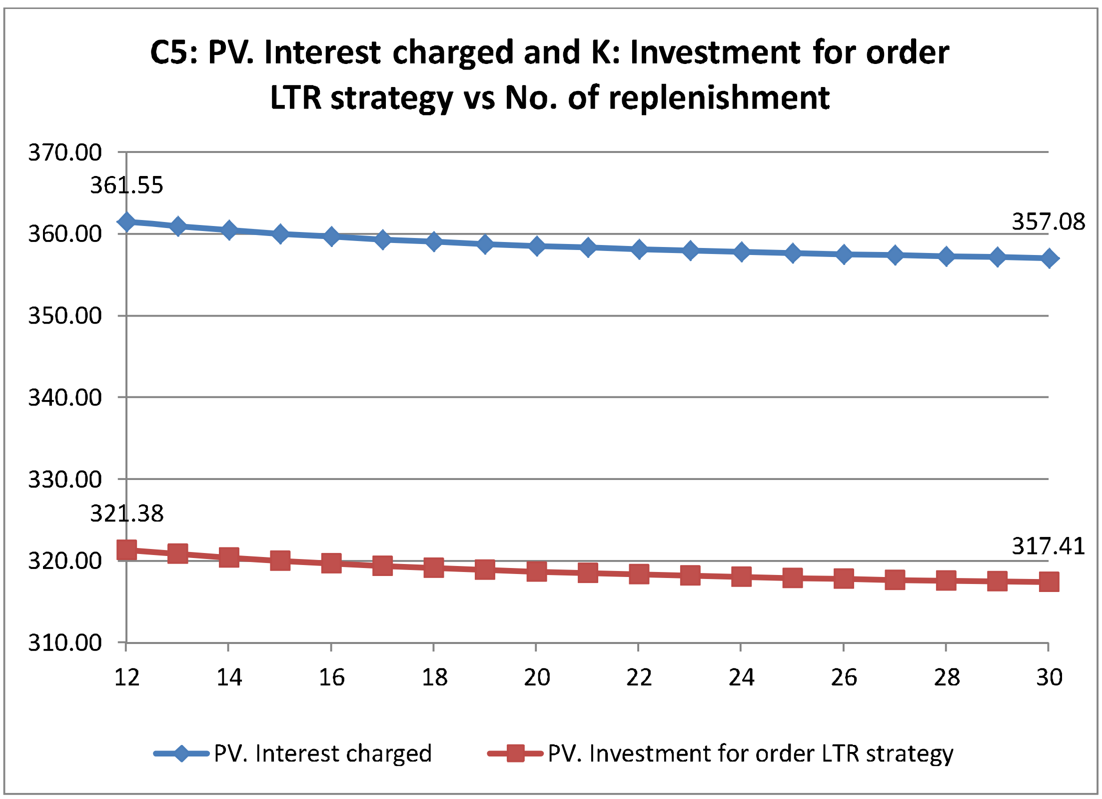


Figure 4-12 C5 and K: Minor cost – PV of interest charged and capital investment for order LTR strategy

Figure 4-11 and Figure 4-12 indicates the minor cost projection for C1, C2, C5 and K over number of replenishment. These costs function is insignificant in the model due to low initial cash value assigned and some do not interact directly with the demand rate variable. However, these low criticality costs function can be improved by looking at their individual trend projection. C2, C5 and K indicate the cost depletion over n is not very significant but C1 shows an aggressive increment of cost over n. This justifies, more shipping cycle requires more capital investment. It can be significant factor if the shipping is expansive due to internal factor such as shipping capacity, large packaging size like shipping a gas turbine engine, heavy item transportation (e.g slab rock, concrete blocks) and external factor such as taxation, legislative policy, currency exchange fluctuation and shipping distance.

4.1.3 **Present value of cost analysis for Joint Total Cost for supplier and buyer**

In making strategic decision of determining the feasible options for economic order quantity, the present value of total cost for supplier and buyer is combined into general cost function as per (30).

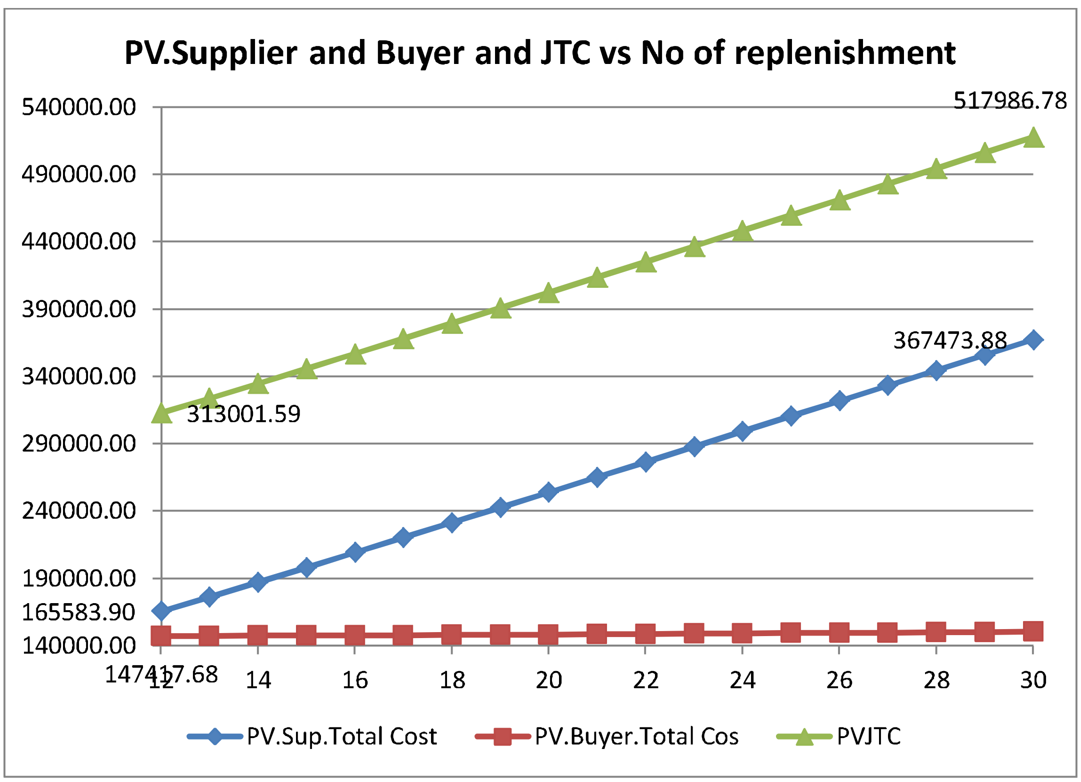


Figure 4-13 Present value of joint total cost in comparison with its' sub-cost function which are the total cost for supplier and buyer.

Figure 4-13 indicates the optimal joint total cost is projected at n =12 for $ 313,001.59. Joint total cost increases as the number of replenishment increases. At optimal n = 12, the economic order quantity for supplier and buyer can be determined. EOQ (supplier) = nDT = 15000 units and EOQ (buyer) = DT = 1250 units respectively. As per Figure 4-2 and Figure 4-8 indicates both parties total cost increasing but supplier total cost is more dominant in driving the joint total cost estimation. This justifies the policy options is going to get more expansive and consideration the optimal number of replenishment is critical to give as much profit for both parties.

4.1.4 **Buyer cost projection with integration of Tripathi, Huang, Hou and Wee models in comparison with the new model**

The computation is done by spreadsheets which is available in the appendix. Four main reference papers for this model is generated to give side by side comparison of in finding the optimal order quantity at the optimum total cost achievable. Most of the papers are only develop the mathematical model based on buyer's point of view.

<u>Tripathi</u>

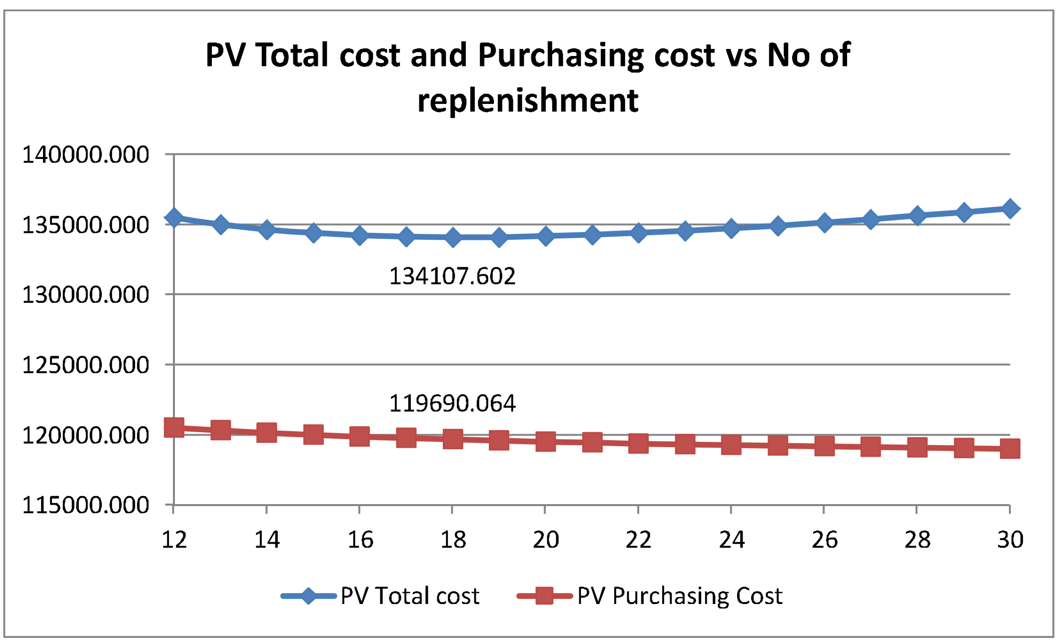


Figure 4-14 PV Total cost and purchasing cost projection integrated with Tripathi's model

In this paper, purchase cost it the major contributor over total cost. It's also correlates with the major cost contributor in this model. Both present value of purchasing cost projection indicates decrement over the number of replenishment as per Figure 4-9. However, the total cost projection yield optimal solution is at n =18 for $134,107.60.

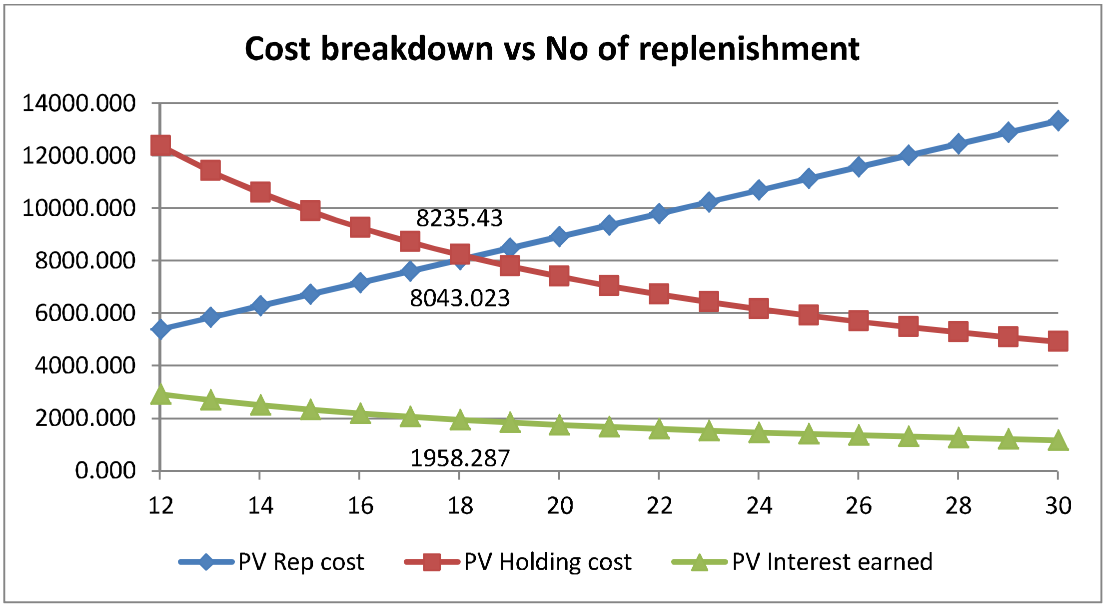


Figure 4-15 PV of cost breakdown projection integrated with Tripathi's model

Figure 4-15 describes the same cost trend projection just as the new model. Certainly the holding cost is decreasing with n because as increasing the inventory cycle over the time horizon will yield lesser inventory holding which contribute to reduction in holding cost (also refer to Figure 4-8).

Tripathi (2010) has simplified his model by using conventional ordering cost compared to the new model which similar to Huang (2010) by employing order processing lead time reduction cost functions in sub equation (29). In comparison for replenishment cost for this and the new model indicates a common trend of cost increment over the planning horizon but it is insignificant in the new model. The replenishment function is refined by incorporating exponential smoothing with investment decision factor which radically reduce the cost breakdown of new model. This justifies the replenishment cost is one of minor cost contributor compared to Tripathi (2010).

The new model and Tripathi (2010) suggests as higher number of replenishment projection shall lead to decreasing interest charged over the planning horizon (also refer to Figure 4-12).

<u>Huang</u>

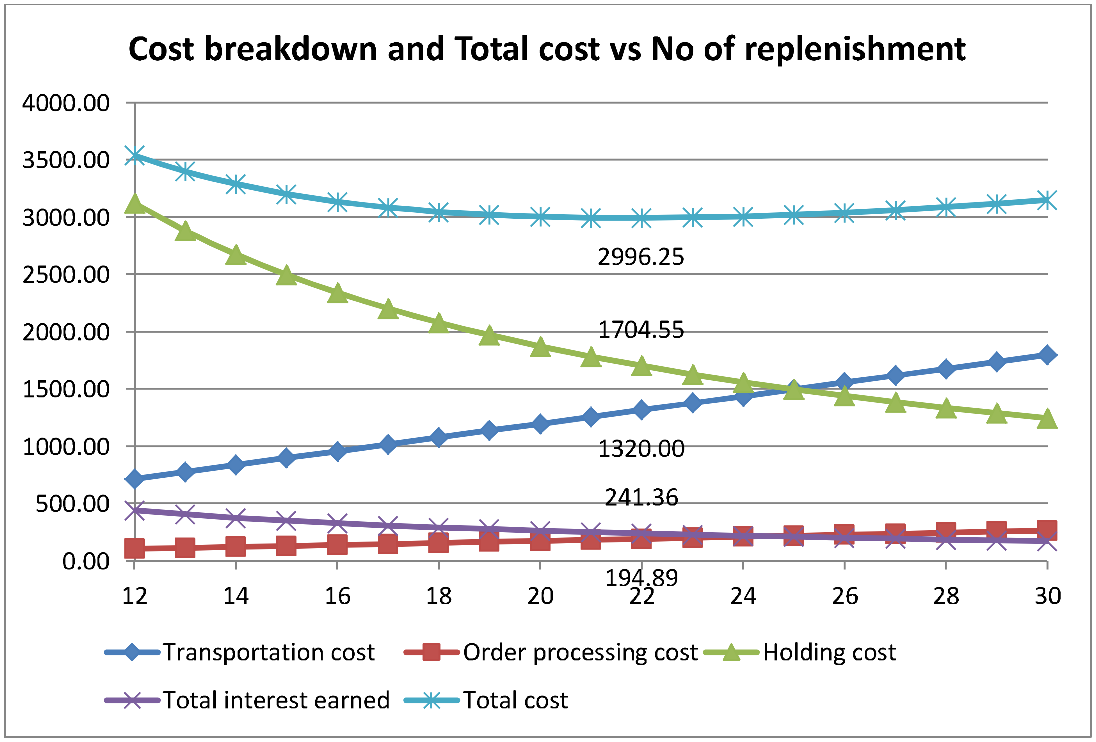


Figure 4-16 Total cost and cost breakdown projection integrated with Huang's model

Huang's (2010) model is the main reference in developing cumulative inventory function for the new model. Huang (2010) also does not consider purchase cost as in new model. In Figure 4-16, holding cost and interest earned are decreased inline with cost projection for holding cost and interest earned for the new model. The transportation cost also present in common with the new model as increasing the number of replenishment, the transportation cost also increases. Over the time horizon, the order processing cost behavior also tally with the new model.

Hou

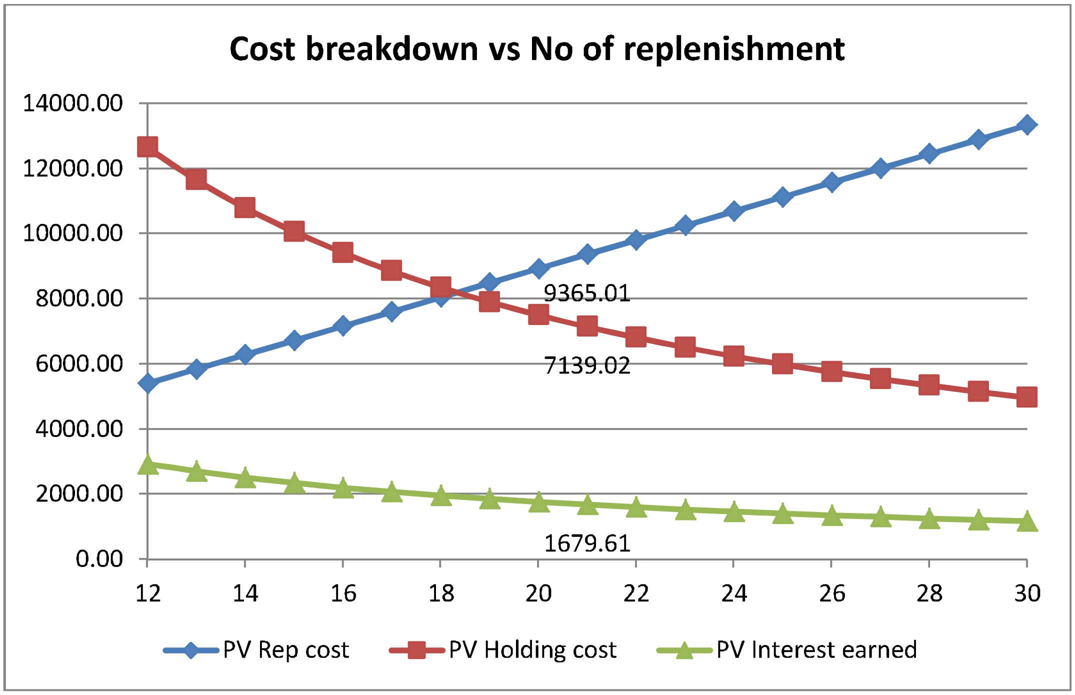


Figure 4-17 PV of cost function breakdown projection integrated with Hou's model

Hou (2009) develop a mathematical inventory cost model based on buyer's point of view. In his model, there are only three cost breakdown presented. As per Figure 4-17, as number of replenishment increase, the replenishment cost also increase. This behavior has in common inline with increasing of order processing cost in new model. So the rest of cost function such as holding cost, interest earned and interest charged (Figure 4-18) are also projected in common with the new model.

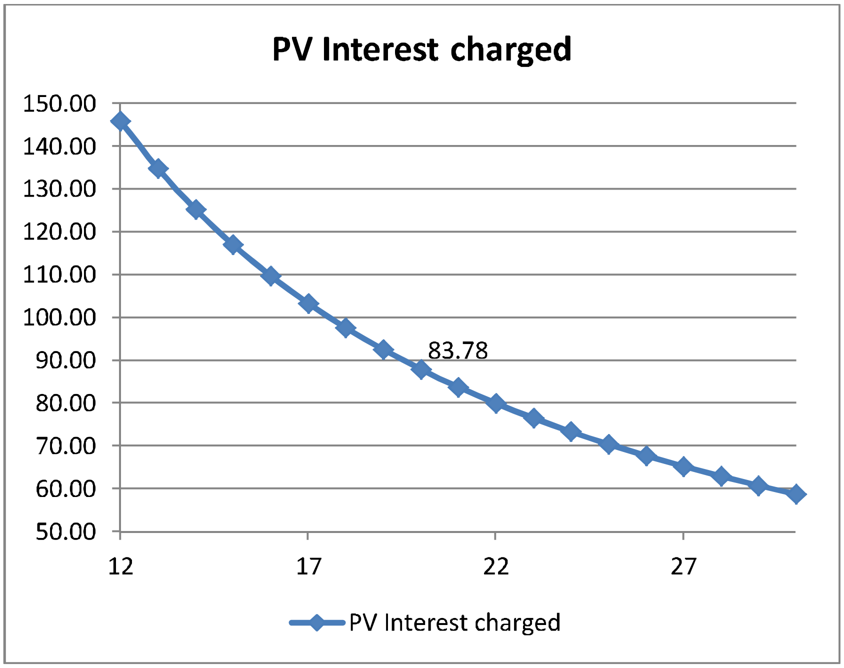


Figure 4-18 PV of Interest charged projection integrated with Hou's model

Wee

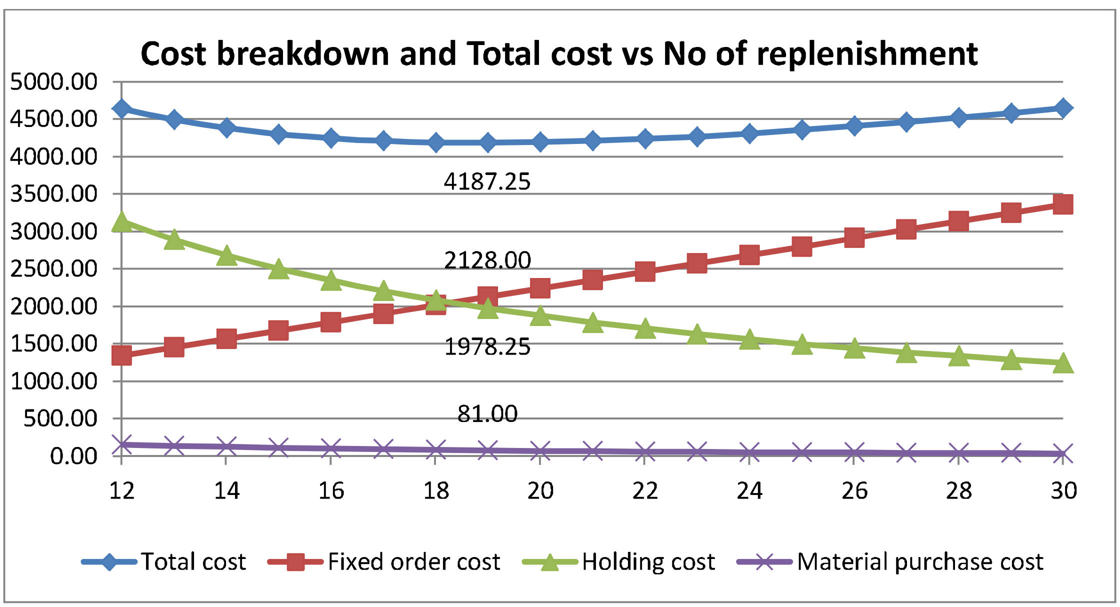


Figure 4-19 Total cost and cost breakdown projection integrated with Wee's model

Wee (1999) follows replenishment cost without LTR as presented in the new model. Thus, the fixed order cost becomes significant increasing over the number of replenishment. The holding cost and the purchase cost suggest in common with the new model as decreasing behavior over the number of replenishment.

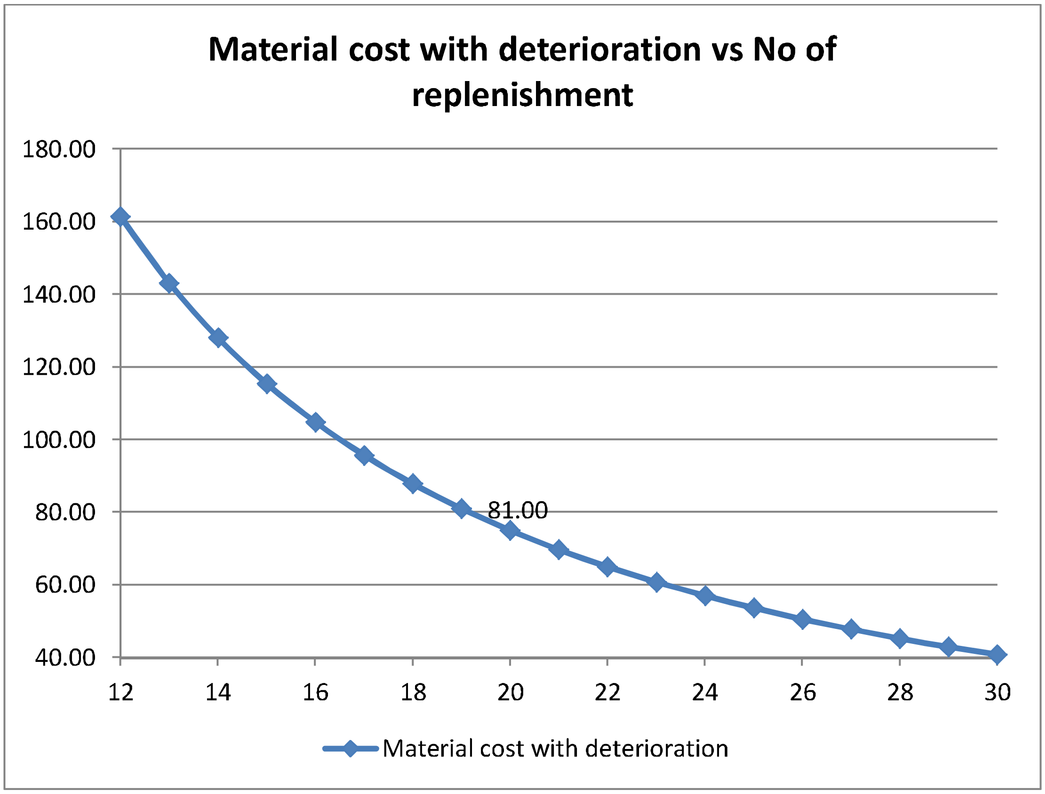


Figure 4-20 Material cost with deterioration projection integrated with Wee's model

4.2 Sensitivity Analysis

4.2.1 **Economic Incentives: Quantity Discount Incentive**

The purpose for analyzing the quantity discount is to determine whether it is feasible for buyer to accept the quantity discount strategy by increasing its economic order quantity by stepsize N. The hypothesis in this analysis is as the percentage of cost deduction is progress higher, more feasible for the buyer accepting the step size of order quantity.

Assume the supplier and buyer has at least 5 years of business partnership and the previous annual demand data is collected for project forecast. The data of previous demand is tabulated in Table 4-1below:

Year	1	2	3	4	5	Mean, G
Demand per cycle (unit)	2000	2400	2300	2800	2500	2400

Table 4-1 Buyer's previous demand database

As the buyer to increase its order quantity, obviously it will subject to overstocking risk. Risk adjustment approach is developed for overstocking cost sharing between supplier and buyer. The proportion of overstocking risk that supplier is willing to bear is defined as Z as discussed in page 29. For estimation case only, sample size of risk factor, Z = 0.8, 0.5, and 0.3 is used.

STEP 1:

Quantity discount schedule is determined by using equation (36). The maximum allowable step size is determined by taking ratio of $N = \frac{EOQ_{Supplier}}{EOQ_{Buyer}}$. EOQ for supplier is 15000 units whereas EOQ for buyer is 1250 units. Thus, yielding the maximum allowable step size = 12.

N	1	2	3	4	5	6	7	8	9	10	11	12
Discount earn, $	0.00	4.78	6.34	7.36	8.38	9.37	10.34	11.28	12.21	13.13	14.03	14.93

Table 4-2 Quantity discount earned from order quantity increment, if Z = 0.8

Table 4-2 indicates the discount earned has surpasses the unit price P =$10 from N = 7 to 12. Thus, the discount schedule only make sense from N = 1 to 6. This because the determination of quantity discount is based on linear ratio of Q(N-1)/NQ. RED regions as in Table 4-2, Table 4-5 and Table 4-8are the non-feasible solutions for the quantity discount strategy.

STEP 2:

Determine the overstocking risk for buyer and percentage of risk sharing using standard normal distribution.

NOTE: STDEV is standard deviation.

If Z = 0.8

N	expected EOQ with step size, N	STDEV	Overstock distribution	Supplier risk distribution	Percentage of risk sharing (bear by client)	Overstock cost with discount
1	1250	1179.1946	16.47%	60.26%	-43.79%	-2065.31
2	2500	279.2848	63.98%	100.00%	-36.02%	-206.50
3	3750	1374.9545	83.69%	97.30%	-13.61%	-274.15
4	5000	2613.0442	84.01%	86.72%	-2.71%	-74.82
5	6250	3858.8211	84.08%	79.38%	4.70%	117.35
6	7500	5106.6623	84.10%	74.81%	9.29%	118.64
7	8750	6355.3521	84.11%	71.80%	12.32%	-106.25
8	10000	7604.4724	84.12%	69.67%	14.45%	-563.69
9	11250	8853.841	84.12%	68.10%	16.02%	-1253.99
10	12500	10103.366	84.13%	66.90%	17.22%	-2176.24
11	13750	11352.995	84.13%	65.95%	18.18%	-3329.40
12	15000	12602.698	84.13%	65.18%	18.95%	-4712.59

Table 4-3 Overstock cost with at supplier sharing factor, Z = 0.8

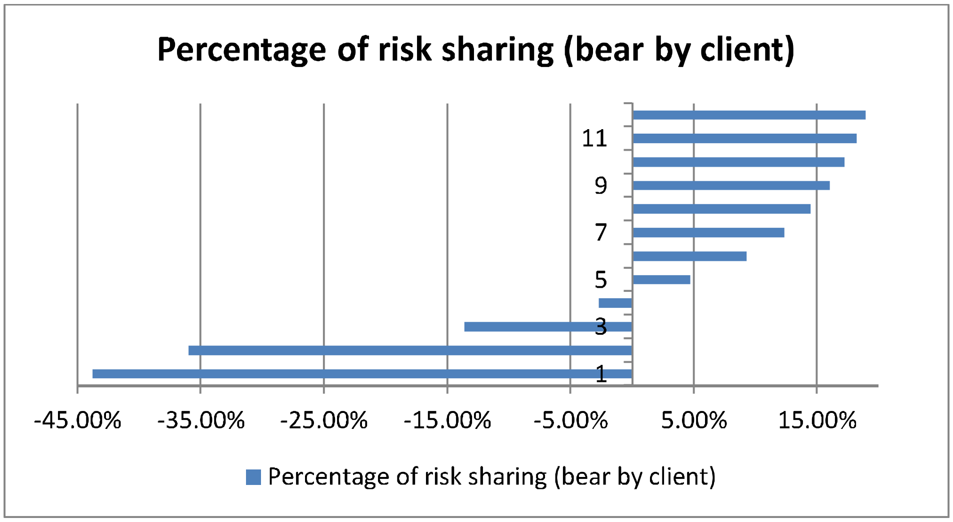


Figure 4-21 Percentage of overstock risk bear by buyer, if Z = 0.8

N	TC w/ step size but No discount, $	TC w/ step size with discount + overstock adjustment, $	Total cost deduction, $	% cost deduction
5	77512.50	13898.37	63514.13	82.05
6	92595.00	7314.80	85280.20	92.10

Table 4-4 Overstock cost with at supplier sharing factor, Z = 0.8

As per Table 4-2the economic justification for buyer is only left for choosing the step size N = 5 and 6. Negative overstock cost in Table 4-3indicates the solution does not favour to supplier. Instead, supplier shall experience loss in revenue as the overstock cost distribution is biased towards the buyer.

On Figure 4-21, left hand side indicates the risk favour to the buyer whereas on the right side indicates the risk favours to the supplier. In determining the optimal solution without sacrificing the supplier in generating profit, the least cost deduction on to buyer is selected in Table 4-4to balance the overstocking cost between supplier and buyer. In this case, buyer may select N = 5 for 6250 units as its' new economic order quantity.

If Z = 0.5

N	1	2	3	4	5	6	7	8	9	10	11	12
Discount	0.00	4.87	6.30	7.08	7.77	8.42	9.02	9.60	10.17	10.72	11.26	11.79

Table 4-5 Quantity discount earned from order quantity increment, if Z = 0.5

N	expected EOQ with step size, N	STDEV	Overstock distribution	Supplier risk distribution	Percentage of risk sharing (bear by client)	Overstock cost with discount
1	1250	1179.1946	16.47%	71.23%	-54.76%	-2582.70
2	2500	279.2848	63.98%	100.00%	-36.02%	-206.50
3	3750	1374.9545	83.69%	98.70%	-15.01%	-305.15
4	5000	2613.0442	84.01%	92.12%	-8.11%	-247.44
5	6250	3858.8211	84.08%	86.86%	-2.78%	-95.45
6	7500	5106.6623	84.10%	83.36%	0.74%	23.89
7	8750	6355.3521	84.11%	80.97%	3.14%	78.12
8	10000	7604.4724	84.12%	79.25%	4.87%	58.65
9	11250	8853.841	84.12%	77.95%	6.17%	-36.76
10	12500	10103.366	84.13%	76.95%	7.18%	-208.56
11	13750	11352.995	84.13%	76.15%	7.98%	-456.61
12	15000	12602.698	84.13%	75.50%	8.63%	-780.63

Table 4-6 Overstock cost with at supplier sharing factor, Z = 0.5

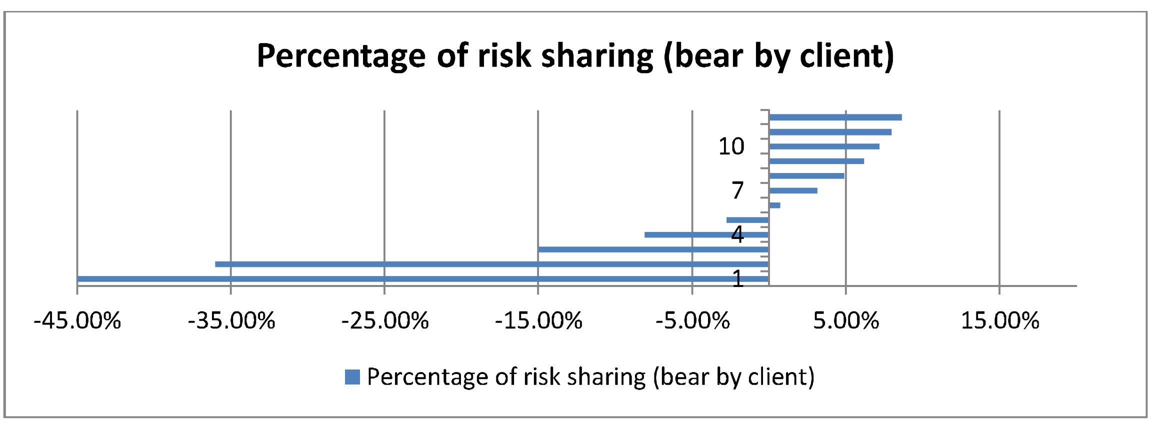


Figure 4-22 Percentage of overstock risk bear by buyer, if Z = 0.5

N	TC w/ step size but No discount, $	TC w/ step size with discount + overstock adjustment, $	Total cost deduction, $	% cost deduction
6	952595.00	15952.25	76642.75	82.77
7	107777.50	11965.03	95812.47	88.90
8	122960.00	6364.44	116595.56	94.82

Table 4-7 Overstock cost with at supplier sharing factor, Z = 0.5

As per Table 4-2 the economic justification for buyer is only left for choosing the step size N = 6, 7 and 8. Still the negative risk percentage occurs in Table 4-6 but the solution provide three feasible options for N at Z = 0.5 rather than Z = 0.8

OnFigure 4-22, left hand side indicates the risk favour to the buyer whereas on the right side indicates the risk favours to the supplier. In determining the optimal solution without sacrificing the supplier in generating profit, the least cost deduction on to buyer is selected in Table 4-7to balance the overstocking cost between supplier and buyer. In this case, buyer may select N = 6 for 7500 units as its' new economic order quantity.

If Z = 0.3

N	1	2	3	4	5	6	7	8	9	10	11	12
Discount earn, $	0.00	4.87	6.29	6.95	7.47	7.91	8.31	8.68	9.04	9.37	9.70	10.02

Table 4-8 Quantity discount earned from order quantity increment, if Z = 0.3

N	expected EOQ with step size, N	STDEV	Overstock distribution	Supplier risk distribution	Percentage of risk sharing (bear by client)	Overstock cost with discount
1	1250	1179.1946	16.47%	77.64%	-61.17%	-2885.09
2	2500	279.2848	63.98%	100.00%	-36.02%	-206.50
3	3750	1374.9545	83.69%	99.24%	-15.55%	-317.15

4	5000	2613.0442	84.01%	94.67%	-10.65%	-339.67
5	6250	3858.8211	84.08%	90.65%	-6.57%	-257.18
6	7500	5106.6623	84.10%	87.87%	-3.77%	-160.91
7	8750	6355.3521	84.11%	85.92%	-1.81%	-77.55
8	10000	7604.4724	84.12%	84.50%	-0.37%	-15.00
9	11250	8853.841	84.12%	83.41%	0.71%	24.25
10	12500	10103.366	84.13%	82.57%	1.56%	39.47
11	13750	11352.995	84.13%	81.89%	2.24%	30.53
12	15000	12602.698	84.13%	81.33%	2.80%	-2.52

Table 4-9 Overstock cost with at supplier sharing factor, Z = 0.3

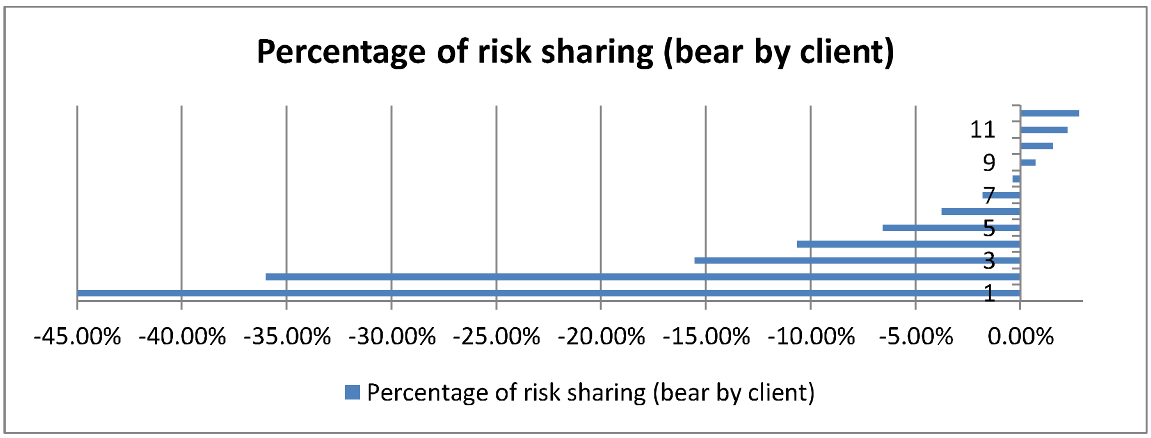


Figure 4-23 Percentage of overstock risk bear by buyer, if Z = 0.3

N	TC w/ step size but No discount, $	TC w/ step size with discount + overstock adjustment, $	Total cost deduction, $	% cost deduction
9	138142.50	14693.80	123448.70	89.36%
10	153325.00	11051.92	142273.08	92.79%
11	168507.50	6543.81	161963.69	96.12%

Table 4-10 Overstock cost with at supplier sharing factor, Z = 0.3

As per Table 4-8 the economic justification for buyer is only left for choosing the step size N = 9, 10 and 11. Still the negative risk percentage occurs in Table 4-9 but the solution provide three feasible options for N at Z = 0.3 but at higher EOQ step size than Z = 0.5 and Z = 0.8.

On Figure 4-23, left hand side indicates the risk favour to the buyer whereas on the right side indicates the risk favours to the supplier. In determining the optimal solution without sacrificing the supplier in generating profit, the least cost deduction on to buyer is selected in Table 4-10to balance the overstocking cost between supplier and buyer. In this case, buyer may select N = 9 for 11250 units as its' new economic order quantity.

In this quantity discount model, there are some limitations highlighted and shall be discussed in the next section.

4.2.2 **Economic incentives: Cash discount on permissible delay in settling trade credit account**

The purpose of this sensitivity analysis is to investigate the effect of permissible delay period to the buyer total cost in the purchase policy at single cycle for particular number of replenishment. The objective is to determine the change of cost functions influenced by various permissible delay periods particularly in the interest charged and interest earned function. The interest charged cost function C5 and the interest earned function is given by:

$$C5 = \frac{D}{2T}(T - t)^2 p \times Ic \quad \text{and} \quad C6 = \frac{D}{2T}(t)^2 p \times Id$$

<u>STEP 1</u>

Set the sample size, t = % x T where (at n = 12, T = 0.4166 years)

y	1	2	3	4	5
%	80	60	40	20	5
t (years)	0.333	0.250	0.167	0.083	0.021

Table 4-11 Variety of permissible delay period, t

<u>STEP 2</u>

Compute the interest charged function C5, interest earned function C6 and buyer total cost function, $TC_b(T,t,K)$ using parameter in Table 4-11 and Table 3-1. Interest charged, I_c = \$ 0.15/\$/year and Interest earned, I_d = \$ 0.12/\$/year.

C5	3.75	15.00	33.75	60.00	84.61
C6	48.00	27.00	12.00	3.00	0.19
TC_b(\$)	16005.17	16030.87	16052.18	16079.08	16100.92

Table 4-12 Sensitivity analysis of interest charged, interest earned and buyer total cost with effect of various trade credit period per inventory cycle.

In Table 4-12 indicates that as supplier decrease the cash incentives by % from left to right, the value of C5 shall increase whereas C6 is decrease.

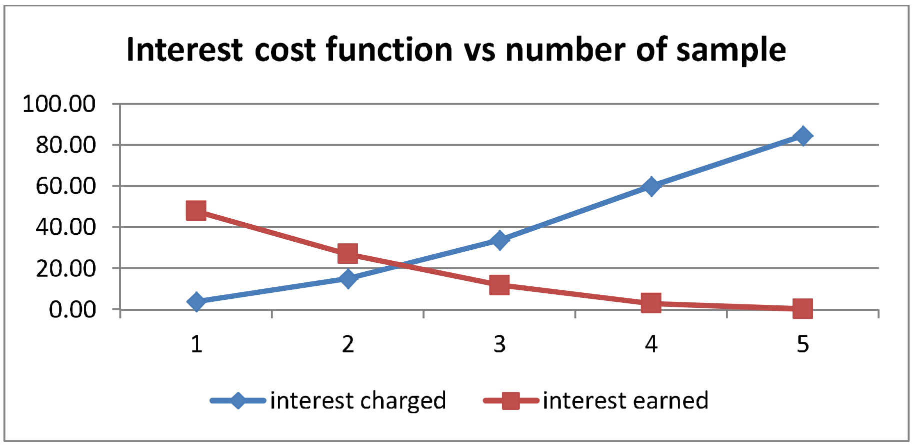


Figure 4-24 Projection of interest charged and interest earned with effect of various trade credit period

Figure 4-24 agreed with the result in Table 4-12 and provide sensitivity against decreasing the supplier cash discount incentive by %.

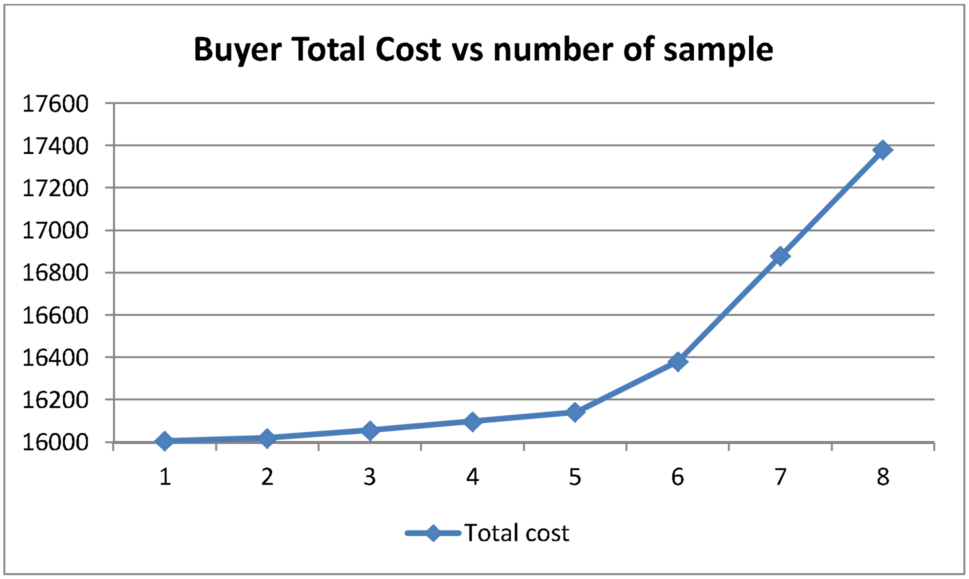


Figure 4-25 Projection of buyer total cost with effect of various trade credit period

Figure 4-25 suggests that as the supplier decreases the permissible delay by % across the table, the buyer total cost increases. This is because as the trade credit becomes smaller, the chance of buyer getting charged by interest is higher whereas only short period of time the buyer can take advantage of interest earned. The rising of buyer total cost across the sample % is because the buyer is will definitely being imposed an interest over its purchased quantity thus contributes to the rising of buyer total cost per cycle. Low cash discount value at short period of t does not favor the buyer but the profit is biased to supplier. Thus the optimal solution is as the supplier willingness to reduce its profit margin by offering higher cash discount at longer period of t. This would contribute to profit sharing among supplier and buyer for long term business partnership.

4.2.3 **Economic factor: Order processing cost lead time reduction**

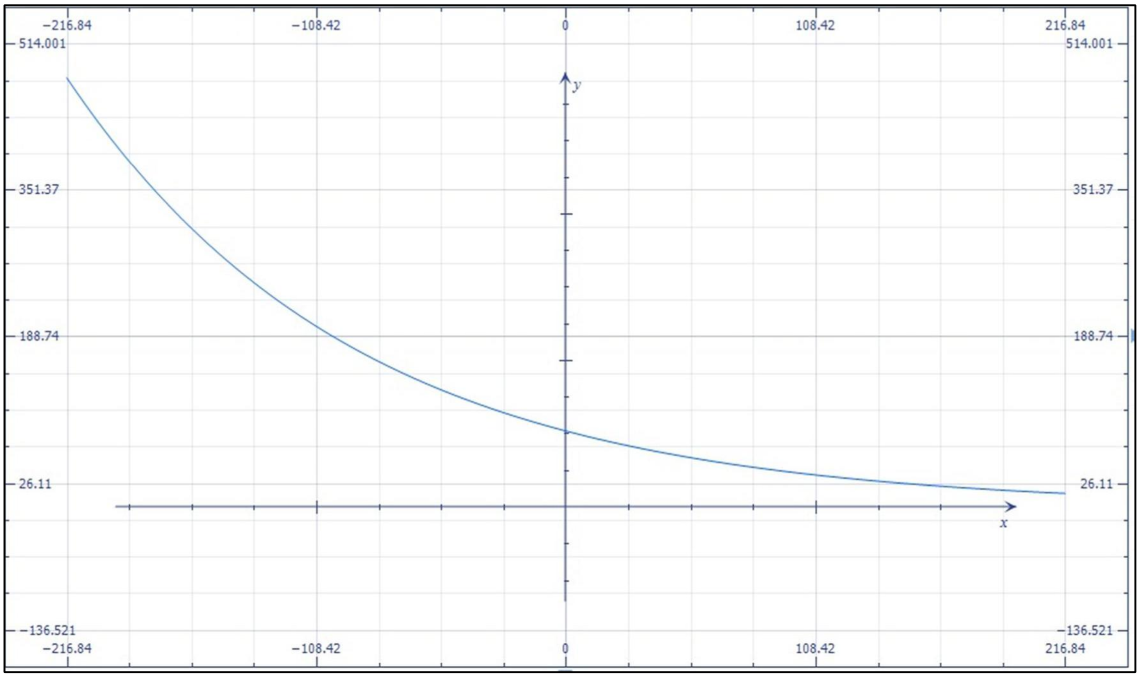


Figure 4-26 Graph of order processing cost with smoothing constant, m = 0.008

Like in Tripathi (2010) analysis, the order processing cost which is replenishment cost is treated as fixed parameter. However, as projected his model in Figure 4-15 indicates that as the processing cost increases as number of replenishment increases. In modern world, many systematic approaches has been implemented in a company to manage their supply chain network such as database system, material resource planning MRP, and enterprise resource planning ERP. With help of these technologies would tremendously cuts the processing time in inventory management and the system would stabilize over the years maintained at its highest efficiency. Thus, smoothing constant of m = 0.008 is favourable to incorporate in the order processing cost function. The order processing cost function is given by:

$$C2 = ULe^{-mK}$$

Huang (2010) Order processing cost function

where U is fixed order processing cost, \$/time, L is average processing period, time/cycle, m is smoothing constant and K is decision factor of capital investment for employing Lead Time Reduction. The total cost for buyer in single cycle period is given by:

$$TC_b(T,t,K) = F + ULe^{-mK} + DTp + \frac{1}{2}DT \times hb + \frac{D}{2T}(T-t)^2 p \times Ic - \frac{D}{2T}(t)^2 p \times Id + K$$

The purpose of this sensitivity analysis is to investigate the effect of decision factor in increment of order processing cost at particular number of replenishment, n = 12. U = $ 560/years, L = 0.15 years/cycle, m = 0.008

STEP 1

Set the sample size, K

j	1	2	3	4	5	6	7	8
$K_{j=1,2,..}$	80	100	150	200	250	500	1000	1500

Table 4-13 Decision factor of capital investment, K ($)

STEP 2

Compute the order processing cost with sample size, K as per

C2	44.29	37.74	25.30	16.96	11.37	1.54	0.03	0.00
TCb($)	16005.17	16018.62	16056.18	16097.84	16142.25	1382.42	16880.91	17380.88

Table 4-14Total cost for buyer without DCF with effect of K

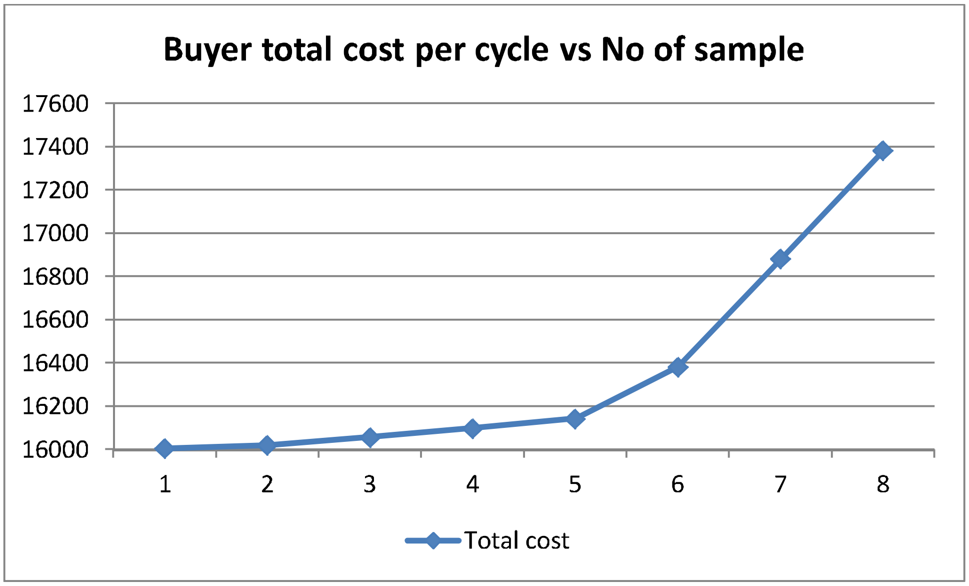


Figure 4-27 Buyer total cost with effect of K sensitivity analysis

In Table 3-1, as per increasing the assigned decision factor K, obviously the Lead Time of processing cost shall significantly decrease and almost approaching Just In Time. However, Figure 4-27 tells different story. The total cost of buyer is actually increasing with decision factor of K. This can be interpreted as the company has invested lots of its capital in reducing the processing lead time but new system to be implemented is expansive and consume more cost and man hour to setup. Thus, for the company to invest heavily in sophisticated inventory management system is not appropriate solution. Furthermore, the order processing cost is a fixed cost overhead which the company to bear over the planning horizon. The equation C2 and K is valid only for explaining the theoretical of balancing the lead time reduction effort and the rise of cost caused by the strategy.

4.3 Limitations

There are some limitations highlighted in each constraint of the new model.

Deterioration factor

- The rate of inventory deterioration can be amplified by increasing the scale factor of α. In this model, the scale factor is set to 1. To represent the real situation, random deteriorating data has to be audited and match with the current Weibull deteriorating function. Since the scale factor is defined earlier during the mathematical development, the sensitivity analysis regarding the deterioration would not be possible as the equations were tailored at particular inventory deterioration behavior.

Net discount of inflation rate factor

- The continuous discounted cash flow present value over the time horizon formula presented by Tripathi (2010) suggested the present value projection of the inventory policy yield dramatic increment of cash value at present. There are incongruency in matching the supplier's inventory function and the buyer inventory function. The buyer total costs is projected at single cycle period whereas supplier's total cost is projected annually but both reaching the same objective which is to estimate the present value of joint total cost under mutual profit sharing and lean supply system.
- The costs breakdown defined in the model has biased towards the buyer. From the previous literature review research, most of mathematical models were developed in buyer's of point of view. There are little knowledge about supplier cost breakdown. However, the projected present value results the policy yields the supplier is the one bear more total costs than buyer.

Permissible delay in settling trade credit account factor

- In this model permissible delay convenience is defined as supplier offer an economic incentive to buyer by setting a credit period in this case, before the end of each cycle period. In actual situation, the sales data from client is unknown to the supplier. Often in real world case where the buyer itself exceed or default their payment and accumulated in the next replenishment cycle.

- The permissible delay period is set constant at $t = 0.15$ year per inventory cycle. More dynamic approach in determining the delay period is required to simulate the actual behavior.

Quantity discount factor

- The determination of quantity discount schedule presented in this model follows the linear factor of Q(N-1)/NQ plus overstocking risk factor E(u). The risk adjustment strategy is employed by allowing the supplier to share overstocking cost to let the buyer increase its economic order quantity stepsize. The demand for the purchase policy has no longer follows a fixed value. A stochastic demand under standard normal distribution is used to determine the probability of overstocking risk carried by buyer if the buyer accepting the quantity discounts policy.
- However, the decision factor Z is set to constant and does give illogical cost sharing solution that favors to the buyer at the same time the supplier shall experience loss in revenue. An extensive approach to let the decision factor dynamically suited with new model is required to make it sensible even though in the sensitivity analysis suggested a few feasible optimal solutions for the buyer in accepting quantity discount policy.
- Solutions in the analysis only valid if the actual demand is less than the new order quantity for the overstock to happen. The quantity discount schedule function includes the overstocking risk sharing. The formula is not valid for order quantity is less than step size demand. Thus, if buyer chooses to stick with its main plan at initial economic order quantity, the quantity discount shall not be employed.

Order processing lead time reduction

- The order processing cost function presented in this model is valid of explaining the theoretical situation where lead time reduction in processing order has to be balanced with its capital investment to implement the LTR policy.
- More significant methodology for this specific factor is required to simulate the actual behavior of order cost function such as breaking cost function into sub-function and analyze the contribution of each sub-function towards the order processing cost.

5 CONCLUSIONS AND RECOMMENDATIONS

Ending Final Year Project 2 (FYP 2), the new mathematical model has been developed by including four economic factors: (1) inflation, (2) quantity discount and (3) permissible delay period (4) order processing lead time reduction and one inventory deterioration factor. The optimal solutions can be determined by analyzing both present value of supplier and buyer total cost in determining the optimal number of replenishment. The next step, is to determine whether it is feasible if the buyer accepting the quantity order policy under overstocking risk sharing between two parties. However, there are some limitations highlighted in this model such as incongruency of projection period between buyer's inventory function and supplier's cumulative inventory functions. Assumptions have been defined earlier to relax some cost functions the mathematical model otherwise it is too difficult to be solved. In the future, this research about regarding to this model can be extended by focusing at the limitations discussed in the previous section.

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5.1 Gantt chart

5.1.1 **Gantt chart for the first semester of final year project**

Table 0-1: Gantt chart for the first semester of final year project

No	Project Activities	Week no.														
		1	2	3	4	5	6	7		8	9	10	11	12	13	14
1	Approval of selected project topic															
2	Preliminary research work															
3	Submission of extended proposal defence															
4	Proposal defence															
5	Project work continues • Metal adsorption experiment commenced															
6	Submission of interim draft report															
7	Submission of interim report															

Process

: Suggested milestone

5.1.2 **Gantt chart for the second semester of final year project**

Table 0-2: Gantt chart for the second semester of final year project

No	Project Activities	Week no.															
		1	2	3	4	5	6	7		8	9	10	11	12	13	14	15
1	Project work continues • Experiment continued • Result analysis																
2	Submission of progress report																

3	Project work continues																
4	Pre-EDX																
5	Submission of draft report																
6	Submission of dissertation (soft bound)																
7	Submission of technical paper																
8	Oral presentation																
9	Submission of project dissertation (hard bound)																

: Process

: Suggested milestone

Integration of new parameter with Tripathi's model

Trade credit period, t	initial inventory, Q	n	T	C1: PV replenishment cost	C2:PV Purchasing cost	A: PV Holding cost	I: PV Interest payable	E: PV Interest earned	PV Total cost	PV Projection
0.333	1250	12	0.417	5399.164	120520.410	12381.76	145.311	2930.636	135516.011	9.64136501
0.308	1154	13	0.385	5839.791	120328.454	11423.22	134.292	2706.650	135019.110	10.4281986
0.286	1071	14	0.357	6280.426	120164.081	10602.43	124.826	2514.469	134657.290	11.2150472
0.267	1000	15	0.333	6721.068	120021.745	9891.67	116.606	2347.770	134403.324	12.0019078
0.250	938	16	0.313	7161.716	119897.293	9270.22	109.402	2201.799	134236.838	12.7887782
0.235	882	17	0.294	7602.368	119787.554	8722.24	103.037	2072.916	134142.287	13.5756566
0.222	833	18	0.278	8043.023	119690.064	8235.43	97.371	1958.287	134107.602	14.3625417
0.211	789	19	0.263	8483.682	119602.881	7800.08	92.296	1855.671	134123.273	15.1494325
0.200	750	20	0.250	8924.344	119524.452	7408.45	87.723	1763.274	134181.699	15.936328
0.190	714	21	0.238	9365.007	119453.522	7054.27	83.583	1679.641	134276.740	16.7232276
0.182	682	22	0.227	9805.673	119389.065	6732.40	79.815	1603.583	134403.375	17.5101308
0.174	652	23	0.217	10246.341	119330.232	6438.63	76.373	1534.114	134557.461	18.2970371
0.167	625	24	0.208	10687.010	119276.320	6169.42	73.215	1470.414	134735.550	19.0839461
0.160	600	25	0.200	11127.680	119226.734	5921.82	70.308	1411.793	134934.748	19.8708575
0.154	577	26	0.192	11568.352	119180.976	5693.32	67.623	1357.667	135152.608	20.6577709
0.148	556	27	0.185	12009.024	119138.617	5481.81	65.135	1307.538	135387.047	21.4446862
0.143	536	28	0.179	12449.698	119099.292	5285.45	62.824	1260.979	135636.281	22.2316032
0.138	517	29	0.172	12890.372	119062.688	5102.66	60.671	1217.621	135898.774	23.0185217
0.133	500	30	0.167	13331.047	119028.530	4932.10	58.661	1177.147	136173.194	23.8054416
			Avg costs	9365.041	119564.364	7607.758	89.951	1809.051		

Integration of new parameter with Huang's model

t	n	T	Q	Transport Cost, F	Order processing cost, U	Buyer holding cost, hb	Intereset charged at T	Interest change during t (Ic - Id)	Interest charged at t	Investment	Buyer total cost	Total interest earned factor
0.333	12	0.417	521	720.00	106.30	3125.00	937.50	120.00	1500.00	33.33	3542.14	442.50
0.308	13	0.385	444	780.00	115.16	2884.62	865.38	110.77	1384.62	30.77	3402.08	408.46
0.286	14	0.357	383	840.00	124.02	2678.57	803.57	102.86	1285.71	28.57	3291.88	379.29
0.267	15	0.333	333	900.00	132.88	2500.00	750.00	96.00	1200.00	26.67	3205.54	354.00
0.250	16	0.313	293	960.00	141.74	2343.75	703.13	90.00	1125.00	25.00	3138.61	331.88
0.235	17	0.294	260	1020.00	150.59	2205.88	661.76	84.71	1058.82	23.53	3087.65	312.35
0.222	18	0.278	231	1080.00	159.45	2083.33	625.00	80.00	1000.00	22.22	3050.01	295.00
0.211	19	0.263	208	1140.00	168.31	1973.68	592.11	75.79	947.37	21.05	3023.57	279.47
0.200	20	0.250	188	1200.00	177.17	1875.00	562.50	72.00	900.00	20.00	3006.67	265.50
0.190	21	0.238	170	1260.00	186.03	1785.71	535.71	68.57	857.14	19.05	2997.93	252.86
0.182	22	0.227	155	1320.00	194.89	1704.55	511.36	65.45	818.18	18.18	2996.25	241.36
0.174	23	0.217	142	1380.00	203.75	1630.43	489.13	62.61	782.61	17.39	3000.70	230.87
0.167	24	0.208	130	1440.00	212.60	1562.50	468.75	60.00	750.00	16.67	3010.52	221.25
0.160	25	0.200	120	1500.00	221.46	1500.00	450.00	57.60	720.00	16.00	3025.06	212.40
0.154	26	0.192	111	1560.00	230.32	1442.31	432.69	55.38	692.31	15.38	3043.78	204.23
0.148	27	0.185	103	1620.00	239.18	1388.89	416.67	53.33	666.67	14.81	3066.22	196.67
0.143	28	0.179	96	1680.00	248.04	1339.29	401.79	51.43	642.86	14.29	3091.97	189.64
0.138	29	0.172	89	1740.00	256.90	1293.10	387.93	49.66	620.69	13.79	3120.69	183.10
0.133	30	0.167	83	1800.00	265.76	1250.00	375.00	48.00	600.00	13.33	3152.09	177.00

Integration of new parameter with Wee’s model

n	T	Lot size, q	Material cost with deterioration	Replenishment cost	Holding cost	Total cost
12	0.417	524	161.37	1344.00	3139.41	4644.78
13	0.385	446	143.12	1456.00	2896.41	4495.53
14	0.357	384	128.06	1568.00	2688.37	4384.43
15	0.333	335	115.47	1680.00	2508.25	4303.72
16	0.313	294	104.82	1792.00	2350.77	4247.58
17	0.294	260	95.70	1904.00	2211.91	4211.62
18	0.278	232	87.84	2016.00	2088.56	4192.40
19	0.263	208	81.00	2128.00	1978.25	4187.25
20	0.250	188	75.00	2240.00	1879.02	4194.02
21	0.238	170	69.71	2352.00	1789.27	4210.98
22	0.227	155	65.01	2464.00	1707.71	4236.72
23	0.217	142	60.82	2576.00	1633.27	4270.08
24	0.208	130	57.05	2688.00	1565.05	4310.10
25	0.200	120	53.67	2800.00	1502.30	4355.97
26	0.192	111	50.60	2912.00	1444.39	4406.99
27	0.185	103	47.81	3024.00	1390.79	4462.60
28	0.179	96	45.28	3136.00	1341.02	4522.29
29	0.172	89	42.95	3248.00	1294.69	4585.64
30	0.167	83	40.82	3360.00	1251.46	4652.28

Integration of new parameter with Hou's model

t	n	T	Q	PV T.Rep cost	PV T.Purchasing cost	PV T.Holding cost	PV Interest payable	PV Interest earned	PV Total cost	PV Series projection
0.333	12	0.417	1290	5399.16	124362.92	12644.24	145.91	2930.53	139621.72	9.64136501
0.308	13	0.385	1188	5839.79	123864.02	11646.43	134.80	2706.56	138778.48	10.42819861
0.286	14	0.357	1101	6280.43	123438.08	10794.55	125.27	2514.39	138123.93	11.21504721
0.267	15	0.333	1025	6721.07	123070.19	10058.79	116.99	2347.70	137619.34	12.00190783
0.250	16	0.313	960	7161.72	122749.26	9416.91	109.74	2201.74	137235.88	12.7887782
0.235	17	0.294	902	7602.37	122466.82	8852.03	103.34	2072.86	136951.69	13.57565661
0.222	18	0.278	851	8043.02	122216.34	8351.08	97.64	1958.24	136749.84	14.36254172
0.211	19	0.263	805	8483.68	121992.69	7903.78	92.54	1855.63	136617.07	15.14943246
0.200	20	0.250	764	8924.34	121791.78	7501.96	87.94	1763.23	136542.80	15.936328
0.190	21	0.238	727	9365.01	121610.31	7139.02	83.78	1679.61	136518.52	16.72322765
0.182	22	0.227	694	9805.67	121445.59	6809.57	79.99	1603.55	136537.28	17.51013084
0.174	23	0.217	663	10246.34	121295.40	6509.19	76.54	1534.08	136593.38	18.29703712
0.167	24	0.208	635	10687.01	121157.90	6234.19	73.37	1470.39	136682.07	19.08394611
0.160	25	0.200	609	11127.68	121031.54	5981.48	70.45	1411.77	136799.38	19.87085746
0.154	26	0.192	585	11568.35	120915.03	5748.46	67.75	1357.64	136941.95	20.65777092
0.148	27	0.185	563	12009.02	120807.26	5532.91	65.25	1307.52	137106.93	21.44468625
0.143	28	0.179	543	12449.70	120707.27	5332.94	62.94	1260.96	137291.89	22.23160324
0.138	29	0.172	524	12890.37	120614.26	5146.92	60.77	1217.60	137494.73	23.01852173
0.133	30	0.167	506	13331.05	120527.52	4973.44	58.76	1177.13	137713.65	23.80544157

New Model (Supplier's total cost)

n	T	exp. projection	Holding cost	Setup cost	Total cost	Joint total cost
12	0.417	9.6414	164780.46	803.45	165583.90	312706.99
13	0.385	10.4282	175531.86	802.17	176334.03	323497.23
14	0.357	11.2150	186413.53	801.07	187214.61	334451.09
15	0.333	12.0019	197394.65	800.13	198194.77	345531.04
16	0.313	12.7888	208453.30	799.30	209252.60	356710.16
17	0.294	13.5757	219573.54	798.57	220372.11	367968.65
18	0.278	14.3625	230743.46	797.92	231541.38	379291.63
19	0.263	15.1494	241954.02	797.34	242751.36	390667.72
20	0.250	15.9363	253198.24	796.82	253995.06	402088.05
21	0.238	16.7232	264470.64	796.34	265266.99	413545.63
22	0.227	17.5101	275766.86	795.92	276562.77	425034.85
23	0.217	18.2970	287083.38	795.52	287878.90	436551.17
24	0.208	19.0839	298417.36	795.16	299212.52	448090.91
25	0.200	19.8709	309766.44	794.83	310561.28	459650.98
26	0.192	20.6578	321128.69	794.53	321923.22	471228.84
27	0.185	21.4447	332502.48	794.25	333296.73	482822.35
28	0.179	22.2316	343886.43	793.99	344680.42	494429.70
29	0.172	23.0185	355279.39	793.74	356073.14	506049.34
30	0.167	23.8054	366680.37	793.51	367473.88	517679.96

New Model (Buyer's cost)

n	T	C1	C2	C3	C4	C5	C6	K	PVTCb
12	0.417	2892.41	427.04	30129.27	120517.06	361.55	7231.02	26.78	147123.09
13	0.385	3128.46	461.89	30081.34	120325.37	360.98	7219.52	24.68	147163.20
14	0.357	3364.51	496.74	30040.31	120161.22	360.48	7209.67	22.89	147236.48
15	0.333	3600.57	531.60	30004.77	120019.08	360.06	7201.14	21.34	147336.26
16	0.313	3836.63	566.45	29973.70	119894.80	359.68	7193.69	19.98	147457.55
17	0.294	4072.70	601.30	29946.30	119785.21	359.36	7187.11	18.79	147596.54
18	0.278	4308.76	636.15	29921.96	119687.85	359.06	7181.27	17.73	147750.25
19	0.263	4544.83	671.01	29900.20	119600.78	358.80	7176.05	16.79	147916.36
20	0.250	4780.90	705.86	29880.62	119522.46	358.57	7171.35	15.94	148092.99
21	0.238	5016.97	740.71	29862.91	119451.63	358.35	7167.10	15.17	148278.64
22	0.227	5253.04	775.57	29846.81	119387.26	358.16	7163.24	14.47	148472.08
23	0.217	5489.11	810.42	29832.13	119328.50	357.99	7159.71	13.84	148672.27
24	0.208	5725.18	845.28	29818.67	119274.66	357.82	7156.48	13.25	148878.39
25	0.200	5961.26	880.13	29806.29	119225.14	357.68	7153.51	12.72	149089.70
26	0.192	6197.33	914.99	29794.86	119179.45	357.54	7150.77	12.22	149305.62
27	0.185	6433.41	949.84	29784.29	119137.15	357.41	7148.23	11.77	149525.63
28	0.179	6669.48	984.69	29774.47	119097.87	357.29	7145.87	11.34	149749.28
29	0.172	6905.56	1019.55	29765.33	119061.32	357.18	7143.68	10.95	149976.21
30	0.167	7141.63	1054.40	29756.80	119027.21	357.08	7141.63	10.58	150206.08